CAIM 2023

The 30th Conference on Applied and Industrial Mathematics

September 14-17, 2023

Iaşi, Romania

e-Book of Abstracts

IAŞI
2023

CAIM 2023

SEPTEMBER 14-17, 2023
Iaşi, ROMANIA

Book of Abstracts

Organized by

"Alexandru Ioan Cuza" University of Iaşi

ROMAI

"Gh. Mihoc- C. Iacob" Institute of Mathematical Statistics and Applied Mathematics of Romanian Academy

AOŞR

Dărmăneşti City Hall

Volume Editor: Stelian Ion

All right reserved to:
Romanian Society of Applied and Industrial Mathematics
Address: ”A.I. Cuza” 13, C438, Craiova, Romania
www.romai.ro
e-mail: romai.board@gmail.com

Scientific Commitee

Vlad Ştefan BARBU (Rouen),
Tudor Barbu (Iaşi),
Constantin BĂCUŢĂ (Delaware),
Viorel BOSTAN (Chişinău),
Cornelia Livia BEJAN (Iaşi),
Vasile BRÂNZĂNESCU (Bucureşti),
Liubomir CHIRIAC (Chişinău),
Sanda CLEJA-ŢIGOIU (Bucureşti),
Dana CONSTANTINESCU (Craiova),
Rodica CURTU (Iowa),
Anca CROITORU (Iaşi),
Gabriel DIMITRIU (Iaşi),
Irinel DRĂGAN (Arlington),
Constantin FETECĂU (Iaşi)
Paul GEORGESCU (Iaşi),
Călin Ioan GHEORGHIU (Cluj-Napoca),
Anca Veronica ION (Bucureşti),
Stelian ION (Bucureşti),
Andrei MOROIANU (Paris),
Sergiu MOROIANU (Bucureşti),
Adrian MUNTEAN (Karlstadt),
Acad. Gabriela MARINOSCHI (Bucureşti),
Costică MOROŞANU (Iaşi),
Lidia PALESE (Bari),
Horia F. Pop (Cluj-Napoca),
Mihail POPA (Chişinău),
Vasile POSTOLICĂ (Bacău),
Liliana RESTUCCIA (Messina),
Adrian Florin RADU (Bergen),
Carmen ROCŞOREANU (Craiova),
Dana SCHLOMIUK (Montreal),
Bianca SATCO (Suceava),
Ioan STANCU MINASIAN (Bucureşti),
Mihaela STERPU (Craiova),
Mirela ŞTEFĂNESCU (Constanţa),
Kiyoyuki TCHIZAWA (Tokyo),
Ioan TODINCĂ (Orleans),
Acad. Ioan TOMESCU (Bucureşti),
Cătalin TRENCHEA (Pittsburgh),
Mihai TURINICI (Iaşi),
Rodica TUDORACHE (Iaşi),
Nicolae VULPE (Chişinău),
Gheorghiţă ZBĂGANU (Bucureşti).

Organizing Commitee:

Vlad Ştefan BARBU
Liubomir CHIRIAC
Constantin FETECĂU
Paul GEORGESCU
Anca Veronica ION
Stelian ION
Carmen ROCŞOREANU
Mihaela STERPU

Local Organizing Commitee: "Alexandru Ioan Cuza" University of Iaşi

President: Costică MOROŞANU
Anca CROITORU
Gabriela TĂNASE
Ana-Maria MOŞNEAGU
Vice-President: Toma CONSTANTIN
Sliviu Dumitru PAVAL
Alex VASILICA
Alin Mihail ADOCHIEI
Cătalin STOICESCU

Contents

Plenary talks

Sharp uncertainty principles and stability results

Cristian Cazacu

Faculty of Mathematics and Computer Science, University of Bucharest, Romania
"Gheorghe Mihoc – Caius Iacob" Institute of Mathematical Statistics and Applied Mathematics of the Romanian Academy
e-mail: cristian.cazacu@fmi.unibuc.ro

In this exposure we present well-known uncertainty principles (motivated by quantum mechanics) or equivalently some special Caffarelli-Kohn-Nirenberg type inequalities applied to either scalar or vector fields. The best constants and extremizers are obtained. In addition, we prove sharp stability results in L^2 for such functional inequalities.

Acknowledgment: *This exposure is based on joint works written in collaboration with Joshua Flynn (McGill University, Montreal, Canada,* email:joshua.flynn@mcgill.ca*) Nguyen Lam (Memorial University of Newfoundland, Canada,* email:nlam@grenfell.mun.ca*) and Guozhen Lu (University of Connecticut, USA,* email:guozhen.lu@ucorn.edu*)*

This talk is partially supported by a grant of the Ministry of Research, Innovation and Digitization, CNCS–UEFISCDI, Romania, project number PN-III-P1-1.1-TE-2021-1539, within PNCDI III.

Regional control strategies applied to biological models. Theoretical results and computational issues

Gabriel Dimitriu

University of Medicine and Pharmacy "Grigore T. Popa",
Department of Medical Informatics and Biostatistics, Iaşi, Romania
e-mail: dimitriu.gabriel@gmail.com

Across the life sciences, we encounter many complex systems over which we wish to exert control. Optimal control is a science of trade-offs; between competing objectives, or in weighing up the benefits of control measures against their costs. We present here theoretical results and computational issues for two biological models to which we apply regional control strategies.

In the first model, we investigate the problem of minimizing the total cost of the damages produced by an alien predator population, and of the regional control paid to reduce this population. The dynamics of the predators is described by a prey-predator system with either local or nonlocal reaction terms. A sufficient condition for the zero-stabilizability (eradicability) of predators is given in terms of the sign of the principal eigenvalue of an appropriate operator that is not self-adjoint, and a stabilizing feedback control with a very simple structure is indicated. We then approach a second model, defined by a two-component reaction-diffusion system to describe the spread of malaria. The model describes the dynamics of the infected mosquitoes and of the infected humans. The spread of the disease is controlled by three actions (controls) implemented in a subdomain of the habitat: killing mosquitoes, treating the infected humans and reducing the contact rate mosquitoes-humans. The problem of the eradicability of the disease is considered, while the cost of the controls is ignored. We prove that it is possible to decrease exponentially both the human and the vector infective population, everywhere in the relevant habitat, by acting only in a suitable subdomain. Later, the regional control problem of reducing the total cost of the damages produced by the disease, and of the intervention in a certain subdomain is treated for the finite time horizon case.

The level set method is a key ingredient for both approaches. An iterative algorithm to decrease the total cost is proposed and numerical results illustrate the effectiveness of the theoretical results.

Mathematica® and Differential Equations

Marian Mureșan

Faculty of Mathematics and Computer Sciences, "Babes-Bolyai" University, Cluj-Napoca, Romania

e-mail: mmarianus24@yahoo.com

The talk is focused on finding an answer to the question: how to realize a strong link between Mathematica and a differential equation, ordinary or partial? A possible answer is offered by the book *Mathematica® and Differential Equations* the base of our exposition.

It is largely recognized that generally it is not easy to solve a differential equation. It is also generally admitted that only a few types of differential equations are solvable in closed- form. Sometimes even the numerical methods are rather difficult to handle. We only mention the stiffness phenomenon.

Mathematica offers a lot of instruments and facilities to approach a solution answering to a differential equation with or without additional conditions. We will discuss how this tool can be used to slove some well known equations like sine-Gordon, Burgers, Poisson, Fisher, Fitzhugh-Nagumo, Korteweg-de Vries, etc.

Differential Equations and Algebras

Mihail Popa

"Vladimir Andrunachievici" Institute of Mathematics and Computer Science, Moldova State University,"Ion Creangă" State Pedagogical University, Chişinău, Republic of Moldova

e-mail: mihailpomd@gmail.com

1. Polynomial differential systems (PDS).

Let us examine the first-order autonomous polynomial system of differential equations (PDS) of the general form

$$\frac{dx^j}{dt} = \sum_{m_i \in \Gamma} P^j_{m_i}(x), (j = 1, 2, \dots, n;\ i = 0, 1, \dots, \ell) \qquad (1),$$

where $\Gamma = \{m_0, m_1, \dots, m_\ell\}$ is a finite set of distinct nonnegative integers, and $x = (x^1, x^2, \dots, x^n)$ is the vector of phase variables with n coordinates. By $N = n \sum_{i=0}^{\ell}(m_i + 1)$ we denote the maximal number of nonzero coefficients of the system (1), and by m_i – degree of homogeneity of polynomial $P^j_{m_i}(x)$ of the system (1) with respect to the coordinates of phase vector x. We will denote such systems by $s^n(\Gamma)$. In the case when $n = 2$, we will write them simply $s(\Gamma)$. Note that all variables and coefficients of the PDS (1) take values from the field of real numbers $\mathbb{R}$.

We will mention that since the set of nonnegative integers is infinite, we can form an infinite set of sets of the type $\Gamma = \{m_0, m_1, \ldots, m_\ell;\ (\ell < \infty)\}$. Therefore, we obtain an infinity of different systems of equations of form (1). And their study from system to system distinguishes cardinal.

These equations represent various mathematical models, for example, from physics, medicine, energetics, biology, and other fields of natural and social sciences. Therefore, their research is very important.

2. The research methods of PDS.

We distinguish two classic research methods: quantitative and qualitative methods. These methods have a shortcoming: they require decreasing of the number N. The stated methods were enriched with a new method, which has its beginning in 1963 in the school of differential equations from Chişinău, Republic of Moldova, under the leadership of the Academician Sibirsky Konstantin (1928-1990). A new research direction was founded, which was later formed as "*The method of algebraic invariants in the theory of differential equations*". This direction included a lot of works by the Academician Konstantin Sibirsky and his disciples, as well as their students [1-5]. The results related to the construction of polynomial bases of invariants and comitants of some classical linear groups (centro-affine – $GL(2,\mathbb{R})$, rotation – $SO(2,\mathbb{R})$ and orthogonal - $O(2,\mathbb{R})$) with the help of which some qualitative properties of autonomous polynomial differential systems and the geometric behavior of their solutions were determined. The mentioned direction was recognized and used in the last decades by specialists from Canada, USA, Brazil, Spain, Slovenia, Belarus, France, Algeria, and some scientific centers of other countries [5].

"*The method of algebraic invariants in the theory of differential equations*" continues to develop today quite effectively in research in the Republic of Moldova and outside of it.

The above-mentioned method has become possible since the 90s of the last century. Due to the following decades, together with this direction, there appeared and developed the research in the fields of invariant processes, Lie algebras and commutative graded algebras of invariants, generating functions and Hilbert series, the theory of orbits, the theory of Lyapunov's stability of unperturbed motion, governed by autonomous polynomial differential systems [6-12]. This direction was consolidated under the name "*Differential equations and algebras*".

The latest theories enriched the method of the Academician K. Sibirsky and made it possible to obtain a series of results in the field of study of systems of type (1) not only for the two-dimensional case ($n = 2$), but also ternary ($n = 3$) and multidimensional ($n > 3$) [10-12].

3. Generating functions.

For the first time, the method of generating functions was used in the investigations of the set of comitants and centro-affine invariants of two-dimensional differential systems $s(\Gamma)$.

The primary generating functions for centro-affine comitants of any system of differential equations $s(\Gamma)$ were constructed. With the help of these functions, the reduced generating functions of the set of comitants and centro-affine invariants were constructed for a series of concrete polynomial differential systems. Using the last generating functions, the type and number of possible elements that form the polynomial bases and algebras of comitants and invariants of these systems were determined [6].

4. Finitely determined graded algebras of comitants S_Γ and invariants SI_Γ, Hilbert series.

It was proved that the sets of comitants and centro-affine invariants of any two-dimensional differential system $s(\Gamma)$ generate, in relation to the unimodular group, commutative graded algebras of infinite dimension but finitely determined (*named by us Sibirsky algebras of comitants and invariants, notated respectively by* S_Γ *and* SI_Γ [6], *who first attracted these comitants and invariants to the research of polynomial differential equations*). It was shown that the reduced generating functions form for these algebras the Hilbert series of these algebras. The tandem of graded algebras and Hilbert series forms a well-developed field of modern algebra. Therefore, graded algebras and Hilbert series in the theory of differential equations enriched this field and gave an impetus to the applications of algebraic methods in the theory of differential equations [6].

5. Lie Algebras.

In the 19th century, the great Norwegian mathematician Sophus Lie (1842-1899) showed that systems of differential equations of the first order admit as the largest algebra of infinitesimal operators (today called Lie operators) the one infinite. This disoriented the specialists in this field to use this device in the research of the mentioned differential systems. But, using the method of algebraic invariants, it was shown that this infinite dimensional algebra for first-order polynomial differential systems of the type (1) can be represented as a direct sum of a Lie algebra of finite dimension, which corresponds to the linear representation of centro-affine group in the space of phase variables and coefficients of given systems, and a Lie algebra of infinite dimension [11]. The new approach with the help of Lie algebras made it possible for any two-dimensional differential system $s(\Gamma)$ of the type (1) to determine the Krull dimensions for Sibirsky algebras of comitants S_Γ and invariants SI_Γ corresponding to these systems, which characterizes the maximal number of algebraically independent invariant elements of the last algebras [6, 9].

6. The Center and Focus Problem.

Using Lie algebras and Sibirsky algebras, it was possible to obtain a quite essential result in the algebraic solution of Poincaré's problem, formulated more than 130 years ago, which is called the Center and Focus Problem [9]. Unusual in this problem is that it was known that from an infinite Poincaré-Lyapunov series of polynomials of coefficients of the system (1) a finite number of them can be extracted, which solves the stated problem. But, even today, this number is not known for any system of the type (1). And this is where Lie algebras and Sibirsky algebras contributed.

Namely, let $N = 2\sum\limits_{i=0}^{\ell}(m_i+1)$ be the maximal possible number of nonzero coefficients of the differential system $s(1, m_1, m_2, \ldots, \ell)$ with polynomial nonlinearities of type (1). Then the number of algebraically independent Poincaré-Lyapunov quantities that participate in solving the Center and Focus Problem does not exceed the number $N-1$ which is equal to the Krull dimension of Sibirsky algebra of comitants for the given system. Also, it is shown that this number can be decreased to $N-3$, which is the Krull dimension of the Sibirsky algebra of invariants for the mentioned system. It is assumed that the number of essential Poincaré-Lyapunov quantities does not exceed $N-1$ and can be improved to $N-3$; and their construction will start with the first algebraically independent nonzero Poincaré-Lyapunov quantities obtained from the Lyapunov identity consecutively up to the mentioned estimates.

7. The theory of orbits.

In the qualitative theory of differential equations of type (1), an important role is attributed to the classification problems of these equations. The classification is carried out according to various principles, for example, by the roots of a characteristic equation, singular points, straight lines or invariant curves, etc. Accordingly to what was said above about Lie algebras, the classification of systems of the type (1) was made according to the dimension of orbits, which is characterized by the modulus of the group, defined by this Lie algebra [6]. In this way, concrete differential systems were classified according to nonsingular invariant varieties (which are located on the orbits of maximal dimension) and singular invariant varieties. This was done for some two-dimensional and ternary differential systems, and invariant integrals were determined for the systems located on the invariant varieties. [6-10].

8. Lyapunov's stability of unperturbed motion, governed by mentioned differential systems.

It is known that the problems of stability or instability of some processes of the surrounding world play an important role in various fields of human activity, for example, in technology, economy, energy, medicine, national security, and other areas of social life. But many of these processes are governed by mathematical models, which are projections of systems of differential equations $s^n(\Gamma)$. Starting in 2016, together with my students, I capitalized on this research, using the methods of Lie algebras and Sibirsky algebras, which until now have not been used in stability problems. This allowed us to obtain new results for the stability of unperturbed motion governed by some systems of the type (1), both two-dimensional and ternary [12], and the mentioned methods to be taken over by other researchers.

9. The contributions made by the team of researchers from the Republic of Moldova to those above mentioned.

Within the above themes, doctoral theses were defended by the following 10 persons: P. Macari (1998), A. Braicov (2002), S. Port (2002), E. Bâcova (2003), E. Staruş (Naidenova) (2004), N. Gherştega (2007), O. Diaconescu (Cerba) (2008), V. Orlov (2013), V. Pricop (2014), N. Neagu (2018). The results obtained by the mentioned researchers under the leadership and with the input of the referent formed the core of the direction "*Differential equations and algebras*". Later, with some works, dr. V. Repeşco [7] as well as full prof. D. Cozma [13] joined this research.

Bibliography

[1] Sibirsky K. S. *The method of invariants in the qualtative theory of differential equations.* Chişinău, RIO AN MSSR, 1968, 184 p. (in Russian).

[2] Sibirsky K. S. *Algebraic invariants of differential equations and matrices.* Chişinău, Shtiintsa, 1976, 268 p. (in Russian).

[3] Sibirsky K. S. *Introduction to the algebraic theory of invariants of differential equations.* Mancester-New York, Mancester University Press 1988. VII, 169 p. (Nonlinear Science: Theory and Applications).

[4] Vulpe N. I. *Polynomial bases of comitants of differential systems and their applications in qualitative theory.* Chişinău, Shtiintsa, 1976, 268 p. (in Russian).

[5] Artes Joan C., Llibre Jaume, Schlomiuk Dana, Vulpe, Nicolae. *Geometric configurations of singularities of planar polynomial differential systems—a global classification in the quadratic case.* Birkhäuser/Springer, Cham, [2021], xii+699 pp.

[6] Popa M. N. *Metode cu algebre la sisteme diferenţiale.* Ed. The Flower Power, Universitatea din Piteşti, Romania, Seria Matematică Aplicată şi Industrială, Nr. 15, 2004, 340 p.

[7] Popa M. N. *Algebre Lie şi sisteme diferenţiale.* Universitatea de Stat din Tiraspol (Chişinău). Tipografia AŞM, 2008, 163 p.

[8] Popa M. N., Repeşco V. F. *Algebre Lie şi sisteme dinamice în plan.* Ministrerul Educaţiei al Republicii Moldova.Universitatea de Stat din Tiraspol (Chişinău). Tipografia UST., 2016, 237 p.

[9] Popa M. N., Pricop V. V. *The Center and Focus Problem: Algebraic Solutions and Hypoteheses.* Ed. Taylor&Frances Group, London, New York 2022, 215 p.

[10] Gerştega N. *Lie algebras for the three-dimensional differential system and Applications.* PhD thesis, Chişinău, 2006, 133 p.

[11] Diaconescu (Cerba) O. *Lie algebras and invariant integrals for polynomial differential systems.* PhD thesis, Chişinău, 2008, 126 p.

[12] Neagu N. *Lie algebras and invariants for differential systems with projections on some mathematical models.* PhD thesis, Chişinău, 2018, 125 p.

[13] Neagu N., Cozma D., Popa M. *Invariant methods for studyng stability of unperturbed motion in ternary differential systems with polynomial nonlinearities*. In: Bukovinian Mathematical Journal, 2016, vol. 4, no. 3-4, p. 133-139.

1. Partial Differential Equations

Determination of some Solutions of the Non-Stationary 2D Navier-Stokes Equations

Iurie Baltag

Technical University of Moldova, Chisinau, Moldova
e-mail: iurie.baltag@mate.utm.md, iubaltag@mail.ru

The following system of partial differential equations is examined:

$$\begin{cases} U_t + \frac{P_x}{\mu} + UU_x + VU_y = \lambda \Delta U \\ V_t + \frac{P_y}{\mu} + UV_x + VV_y = \lambda \Delta U \\ U_x + V_y = 0 \end{cases} \tag{1}$$

$P = P(t,x,y), u = u(t,x,y), v = v(t,x,y); U_x = \frac{\partial U}{\partial x}; \Delta U = U_{xx} + U_{yy}; t \geq 0; x, y \in R;$

System (1) describes the process of plane flow of a liquid or gas; this system represents the Navier-Stokes equations in the case of bidimensional non-stationary motion of a viscous incompressible fluid.

The P function represent the pressure of the liquid, and U, V functions represent the flow of the liquid or gas. The constants $\lambda > 0$ and $\mu > 0$ are a determined parameter of the studied liquids or gas viscosity and density. We mention here that $\lambda = \frac{c}{R_e}, c > 0$, where R_e is the Reynolds number.

A number of solutions to the stationary equations have been determined in the papers [1], [2].

In this paper we will the solutions of system (1) are searched for in the following form:

$$U = a(t)u(x,y), V = \beta v(x,y), P = \gamma(t)p(x,y). \tag{2}$$

Applying the variable separation method, from the condition $U_x + V_y = 0$ we determine, that

$$u_x = Cv_y, \quad and \quad \beta(t) = -C\alpha(t), where\ C \quad is\ non-zero\ constant. \tag{3}$$

Substituting relations (3) into the first two equations of system (1) and applying the method set forth in [1], to the proof of theorem 1, we obtain:

$$\alpha'(u_y + Cv_x) + \alpha^2 \left[\left(u\left(u_y + Cv_x\right)\right)_x - C\left(v\left(u_y + Cv_x\right)\right)_y \right] = \alpha\lambda\Delta\left(u_y + Cv_x\right). \tag{4}$$

And, assuming that $\gamma(t) = c \cdot \alpha^2(t)$, where c is a non-zero constant, we get:

$$\frac{\alpha'}{\alpha} = -\frac{\alpha}{u}\left[G_x - Cv\left(u_y + Cv_x\right)\right] + \frac{\lambda \Delta u}{u} = \frac{\alpha}{Cv}\left[G_y - u\left(u_y + Cv_x\right)\right] + \frac{\lambda \Delta v}{v}, \tag{5}$$

where $G = \frac{c}{\mu}p + \frac{1}{2}\left(u^2 + C^2v^2\right)$. The following theorem holds:

Theorem 1. Suppose $\alpha(t)$ is a differentiable function for $t \geq 0$ and let that in the connected domain D the functions $u(x,y)$, $v(x,y)$ and $p(x,y)$ admit continuous partial derivatives up to and including the second order. And $f(z) = u + iCv$ is a differentiable function of complex variable $z = x + iy$. Then system (1) has the following solutions:

$$U = \alpha(t)u(x,y), V = -C\alpha(t)v(x,y); u = Ref, v = \frac{Imf}{C}; \alpha(t) = \frac{1}{C_2 - C_1 t}$$

$$P = c\alpha^2(t)p(x,y), p = \mu\left[C_0 - 0,5\left(u^2 + (Cv)^2 + C_1\left(\int cvdx - \int udy\right)\right)\right],$$

where $c \neq 0$, $C \neq 0$, C_1, C_2, C_0 are arbitrary constants.

If the conditions of theorem 1 are met, then $u_y = -Cv_(x)$.

Next, the case when $u_y \neq -Cv_(x)$. The following theorem gives us solutions in which the viscosity parameter participates explicitly.

Theorem 2. Suppose $\alpha(t)$ is a differentiable function for $t \geq 0$ and let that in the connected domain D the functions $u(x,y)$, $v(x,y)$ and $p(x,y)$ admit continuous partial derivatives up to and including the second order. Then system (1) has the following solutions:

$U = \alpha(t)u(x,y)$, $V = -C\alpha(t)v(x,y)$; $\alpha(t) = C_0e^{C_2t}$; $u = a_1(x)b_1(y)$, $v = a_2(x)b_2(y)$, where the mentioned functions are solutions of the following equations:

$$a_n'' = \frac{c_3}{\lambda}a_n, b_n'' = \frac{c_2 - c_3}{\lambda}b_n, n = 1,2; a_1' = C_3a_2, Cb_2' = C_3b_1$$

$$P = c\alpha^2(t)p(x,y), p = \mu\left[C_0 - 0,5\left(u^2 + (Cv)^2 + C_2\left(\int Cv(u_y + Cv_x)dx + \int u(u_y + Cv_x)dy\right)\right)\right].$$

An example of the application of this theorem if $C_3 < 0$, $C_2 < C_3$ is the following solution of system (1):

$$U = C_0e^{C_2t}(a\cos(kx) + b\sin(kx))\frac{sC}{C_3}(r\cos(su) - d\sin(su));$$

$$V = -CC_0e^{C_2t}\frac{k}{C_3}(b\cos(kx) - a\sin(kx))(dcos(su) + rsin(su)); k^2 = \frac{-C_3}{\lambda}, s^2 = \frac{C_3 - C_2}{\lambda};$$

$$P = cC_0^2e^{2C_2t}\mu\left[C_0 - 0,5\left(u^2 + (Cv)^2 + C_2\left(\int Cv(u_y + Cv_x)dx + \int u(u_y + Cv_x)dy\right)\right)\right].$$

Here $C \geq 0$, $C_0 \geq 0$, a, b, d, r are arbitrary constants.

Some solutions of the examined system are obtained in the paper [3] by applying other research methods.

Bibliography

[1] Baltag Iu. *Determination of some solutions of stationary 2D Navier-Stokes equations*, Journal of Engineering Science, Chisinau, 2022, Vol XXIX (4), pp. 38-50,ISSN 2587-3474, e ISSN 2587-3482. https://doi.org/10.52326/jes.utm.2022.29(4).03

[2] Baltag Iu. *The determination on some exact solutions of the stationary Navier-Stokes equations*, Acta et Commentationes, Exact and Natural Sciences, Chisinau, 2022, Vol. 14, No 2, pp. 106-116, https://revista.ust.md/index.php/acta exacte/article/view/820

[3] Koptev A. V. *Generator of Solutions for 2D Navier-Stokes Equations*, Journal of Siberian Federal University. Mathematics and Physics 2014, 7(3), 324–330,

Spectral Collocation Solutions to some Linear Pseudo-parabolic Problems

Călin-Ioan Gheorghiu

"T. Popoviciu" Institute of Numerical Analysis of Romanian Academy, Cluj-Napoca, Romania
e-mail: ghcalin@ictp.acad.ro

We are concerned with non-periodic *spectral collocation solutions* for *initial-boundary value problems* attached to *linear pseudo-parabolic (Sobolev) equations* of the form

$$M(u_t) = L(u) + g(x,t), x \in (-1,1), t > 0,$$

where the linear part is of the form $L(u) := c(t)\mathrm{i}^{m+1}(\partial^m u/\partial x^m)$ but more general *dispersive terms* are also treatable and the second order operator M is elliptic. The real function $c(t)$ is often a constant and the forcing term $g(x,t)$ can be embedded into $L(u)$.

The Korteweg-de Vries (KdV), Benjamin Bona Mahony (BBM), mod- ified BBM and Benjamin Bona Mahony-Burgers (BBM-B) and nonlinear Schr¨odinger (NLS) initial-boundary value problems can be simplified (lin-earized) to such problems. Actually they contain a nonlinear term N(u) which can be of the form $u^n u_x, n = 1, 2, ...$ and can exhibit some Hamilto-nian invariants.

Chebyshev collocation in the conventional form or using Chebfun is used in order to discretize.

In order to march in time we use several one step and linear multistep FD schemes such that the method of lines (MoL) involved is *stable* in sense of Lax. In order to prove the *numerical stability* we use the *pseudospectra* of the linearized spatial discretization operators of (KdV), (BBM), (NLS), etc..

The effectiveness of our approach has been confirmed by some numerical experiments on the following problem:

$$\begin{cases} \rho u_t = \dfrac{a}{2}u_{xx} + cu_{xxt}, 0 < x < h, t > 0, \\ u(0,t) = u(h,t) = 0, t \geq 0, \\ u(x,0) = 2kx(x-h)/d, 0 \leq x \leq h, k < 0; \end{cases}$$

where ρ, a, d and$c$ are positive constants.

Parabolic PDE in Mass Transfer problems and their solution by Laplace Transform

Victor Martinez-Luaces

University of the Republic of Uruguay,
University of Granada, Spain
e-mail: victorml@correo.ugr.es

Mass Transfer problems are usually modeled using Fick's Law, which gives rise to parabolic PDEs, which often admit to be solved by Laplace Transform.
In this paper three of these problems are analyzed, two of them related to Food Technology Engineering (drying of a vegetable cut into slices and diffusion of sugar in a fruit considered as spherical) and the other one, linked to Chemistry and Chemical Engineering (diffusion in a dissolved

oxygen electrode). In all these problems, parabolic PDEs, with different initial and boundary conditions are considered. The common element is their solution by the Laplace Transform in the time variable.
The method can be extended to other similar problems that are only briefly mentioned, since they have been already described in previous works.

Keywords: parabolic PDE, mass transfer, Laplace transform

On a Local and Nonlocal Second-Order Anisotropic Reaction-Diffusion model with in-homogeneous Cauchy-Neumann boundary conditions: Applications to Image Processing

Costică Moroșanu

"Al. I. Cuza" University, Iaşi, Romania
e-mail: costica.morosanu@uaic.ro

Some reaction-diffusion models, with local or nonlocal diffusion and cubic nonlinear reaction term, endowed with in-homogeneous Cauchy-Neumann (Robin) boundary conditions, we are addressing in the present work. First, we study the existence of the solution in $C((0,t^*],C^1(\Omega))$ of the nonlocal and nonlinear second-order boundary value problem. Next, under certain assumptions on the input data, we prove the well-posedness of the clasical solution in the Sobolev space $W_p^{1,2}(Q)$ of the local and nonlinear second-order boundary value problem.

Using finite difference method, we derive two explicit numerical schemes in order to approximate the unique solution of a particular mathematical model (local and nonlocal case). To illustrate the performance of our new theoretical models, we present some numerical experiments by applying them to the image segmentation tasks.

Bibliography

[1] Miranville A. and Moroşanu C., Qualitative and quantitative analysis for the mathematical models of phase separation and transition. Aplications, *AIMS - American Institute of Mathematical Sciences*, Differential Equations & Dynamical Systems, **7**(2020), www.aimsciences.org/fileAIMS/cms/news/info/28df2b3d-ffac-4598-a89b-9494392d1394.pdf,

[2] Miranville A. and Moroşanu C., *A Qualitative Analysis of a Nonlinear Second-Order Anisotropic Diffusion Problem with Non-homogeneous Cauchy–Stefan–Boltzmann Boundary Conditions*, Appl. Math. Optim., **84**(1) (2021), 227-244, https://doi.org/10.1007/s00245-019-09643-5

[3] Moroşanu C., Analysis and optimal control of phase-field transition system: Fractional steps methods, *Bentham Science Publishers*, 2012, https://dx.doi.org/10.2174/97816080535061120101

On a nonlocal and nonlinear second-order anisotropic reaction-diffusion system with in-homogeneous Cauchy-Neumann boundary conditions. Applications on epidemic infection spread

Cătălin Stoicescu

Department of Computer Engineering,
"Gheorghe Asachi" Technical University of Iaşi
e-mail: stoicescu.catalin@gmail.com

In our current paper we are following the results obtained by Pavăl et al. in [3] and study a nonlocal form of the system they propose. First we are performing a qualitative analysis (see [1], [2] and references therein) for the equivalent non-local second-order system of coupled PDEs, equipped with nonlinear anisotropic diffusion and cubic nonlinear reaction. Our PDEs system is also implementing a SEIRD (Susceptible, Exposed, Infected, Recovered, Deceased) epidemic model. In order to be able to compare with the before mentioned results we use the same hypothesis on the input data: $S_0(x)$, $E_0(x)$, $I_0(x)$, $R_0(x)$, $D_0(x)$, $f(t,x)$ and $w_i(t,x), i = 1,2,3,4,5$, and we prove the well-posedness of a classical solution in $C((0,T], C^1(\Omega))$, extending the types already proven by other authors.

Secondly we construct the implicit-explicit (IMEX) numerical approximation scheme which allows to compute the solution of the system of coupled PDEs. The results are then compared with the ones obtained by [3].

Bibliography

[1] A. Miranville and C. Moroşanu, Qualitative and quantitative analysis for the mathematical models of phase separation and transition. Aplications, *AIMS - American Institute of Mathematical Sciences*, Differential Equations & Dynamical Systems, **7** (2020). www.aimsciences.org/fileAIMS/cms/news/info/28df2b3d-ffac-4598-a89b-9494392d1394.pdf,

[2] A. Miranville and C. Moroşanu, A qualitative analysis of a nonlinear second-order anisotropic diffusion problem with non-homogeneous Cauchy-Stefan-Boltzmann boundary conditions, *Appl. Math. Optim.*, **84** (2021), 227-244.

[3] S. D. Pavăl, A. Vasilicvă and A. Adochiei, Qualitative and quantitative analysis of a nonlinear second-order anisotropic reaction-diffusion model of an epidemic infection spread, *Discrete and Continuous Dynamical Systems - S*, 2022.

A first-order fractional steps type method to approximate a nonlinear reaction-diffusion equation with homogeneous Cauchy-Neumann boundary conditions

Gabriela Tănase

"Al. I. Cuza" University, Iaşi, Romania
e-mail: gabriela.tanase@uaic.ro

The paper concerns with the approximation of solutions to the nonlinear reaction-diffusion equation, endowed with homogeneous Cauchy-Neumann boundary conditions. It extends the studied types of boundary conditions, already proven by other authors, which makes the problem to be more able to describe many important phenomena of two-phase systems, in particular, the interactions with the walls in confined systems. The convergence and error estimate results for a new iterative scheme of fractional steps type, associated to the nonlinear parabolic problem, are also established. The advantage of such method consists in simplifying the numerical computation. On the basis of this approach, a conceptual numerical algorithm is formulated in the end.

Bibliography

[1] T. Benincasa, A. Favini and C. Moroşanu, *A Product Formula Approach to a Nonhomogeneous Boundary Optimal Control Problem Governed by Nonlinear Phase-field Transition System. PART II: Lie-Trotter Product Formula,* J. Optim. Theory and Appl., vol. **148**(1), p. 31-45, 2011.

[2] A. Miranville and C. Moroşanu, Qualitative and quantitative analysis for the mathematical models of phase separation and transition. Aplications, Volume 7; AIMS - American Institute of Mathematical Sciences, Differential Equations & Dynamical Systems, 2020. www.aimsciences.org/fileAIMS/cms/news/info/28df2b3d-ffac-4598-a89b-9494392d1394.pdf,

[3] Moroşanu C., Analysis and optimal control of phase-field transition system: Fractional steps methods, *Bentham Science Publishers*, 2012, https://dx.doi.org/10.2174/97816080535061120101

2. ODEs; Dynamical Systems

On the dynamics of a class of differential systems with two limit cycles

Rachid Boukoucha

Department of Technology, Faculty of Technology, University of Bejaia, 06000 Bejaia, Algeria
e-mail: rachid.boukoucha@univ-bejaia.dz

An important problem of the qualitative theory of differential equations is to determine the limit cycles of a system of the form

$$\begin{cases} x' = \dfrac{dx}{dt} = P(x,y), \\ y' = \dfrac{dy}{dt} = Q(x,y), \end{cases} \tag{1}$$

where $P(x,y)$ and $Q(x,y)$ are real polynomials in the variables x and y. Here, the degree of system (1) is denoted by $n = \max\{\deg P, \deg Q\}$. In 1900 Hilbert [1] in the second part of his 16th problem proposed to find an estimation of the uniform upper bound for the number of limit cycles of all polynomial vector fields of a given degree, and also to study their distribution or configuration in the plane $\mathbb{R}^2$. This has been one of the main problems in the qualitative theory of planar differential equations in the 20th century.

In this work, we consider the special class family of the polynomial differential system of the form (1). We give an explicit expression of invariant algebraic curves, then we prove that these systems are integrable and we introduce an explicit expression of a first integral. Moreover, we determine sufficient conditions for a polynomial differential system (1) to possess two limit cycles, explicitly given. Concrete examples exhibiting the applicability of our result are introduced.

Keywords: dynamical system; Hilbert 16th problem; limit cycle; invariant algebraic curve; first integral.

Bibliography

[1] D. Hilbert, *Mathematische Probleme*; English transl, Bull. Amer. Math. Soc., Volume 8, pp. 437-479, (1902).

[2] R. Boukoucha, *Explicit limit cycles of a family of polynomial differential systems*; Elect. J. of Diff. Equ., Volume 217, pp. 1-7, (2017).

[3] S. E. Hamizi and R. Boukoucha, *A class of planar differential systems with explicit expression for two limit cycles*, Siberian Electronic Mathematical Reports Vol. 17 (2020), pp. 1588-1597.

The study of the behavior of a three-dimensional system with cubic perturbation

Maria-Liliana Bucur

Department of Applied Mathematics, University of Craiova, Romania
e-mail: lilianabucur@yahoo.com

In this paper we will study the stability of equilibrium points of the three-dimensional system:

$$\begin{cases} \dot{x} &= y \\ \dot{y} &= z \\ \dot{z} &= k^2\alpha x^2 + kx - 2\alpha x^2 y - kz \end{cases}$$

which comes from a type jerk equation.

For this system we will study the stability of the equilibrium points and Hopf bifurcation.

Keywords: jerk equations; stability; bifurcations.

On the extreme multistability of some 3D dynamical systems

Dana Constantinescu

University of Craiova, Romania
e-mail: constantinescu.dana@ucv.ro

The multistability (the coexistence of many attractors for the same set of parameters values) is an important property of many dynamical systems. The extreme multistability arises when infinitely many attractors coexist for a given set of parameters values. Although it seems rather exotic, extreme multistability is an important feature of some fundamental models in chemistry, physics, ecology, neuroscience. For now there is no systematic study of the systems that exihibit extreme stability, most of the efforts being directed towards the analysis of 6D dynamical systems obtained by coupling two oscillators.

In this work we study the extreme multistability of three 3D dissipative dynamical systems with an infinite number of equilibria situated on a line. For each system we find a conserved quantity that have a specific value for each equilibrium point. Consequently, we observe that the phase space is foliated into infinitely many manifolds with at least one equilibrium existing in each manifold. We consider also some small perturbations of the systems and we analyze the perturbed systems using the singular perturbation theory.

A new population model with pseudo exponential survival

Dragoș Pătru Covei[1], Traian Pîrvu[2] and Cătălin Șterbeți[3]

[1]*Department of Applied Mathematics, The Bucharest University of Economic Studies, Bucharest, Romania*
[**]*Department of Mathematics and Statistics, McMaster University, Hamilton, Canada*
[3] *Department of Applied Mathematics, University of Craiova, Craiova, Romania*
e-mail: coveidragos@yahoo.com, traian_a_pirvu@yahoo.com, sterbetiro@yahoo.com

The paper considers a model for population dynamics with age structure. The probability of survival here is assumed to be a linear combination of exponentials. The number of births in unit time is characterized through a system of ordinary differential equations. This is solved explicitly in special cases.

Keywords Dynamical systems, Population Dynamics, Differential-difference equations, Laplace transform.

Center conditions for a cubic differential system having an invariant conic

Dumitru Cozma

Ion Creangă State Pedagogical University, Republic of Moldova
e-mail: dcozma@gmail.com

We consider the cubic system of differential equations

$$\dot{x} = y + p_2(x,y) + p_3(x,y),\ \dot{y} = -x + q_2(x,y) + q_3(x,y), \tag{1}$$

where $p_j(x,y),\ q_j(x,y) \in \mathbb{R}[x,y]$ are homogeneous polinomials of degree j. The origin $O(0,0)$ is a singular point for (1) with purely imaginary eigenvalues, i.e. a focus or a center. The problem of distinguishing between a center and a focus (the problem of the center) is open for general cubic systems.

The problem of the center was solved for cubic system (1) with: four invariant straight lines, three invariant straight lines, two parallel invariant straight lines, two invariant straight lines and one irreducible invariant conic, two invariant straight lines and one invariant cubic ([1], [2], [3], [4]).

By means of constructing integrating factors, the center conditions were obtained for a cubic differential system (1) with: one invariant straight line and one invariant cubic curve in [5], one invariant cubic curve in [6].

In this talk, we discuss the problem of the center for cubic differential system (1) with algebraic solutions and analytic integrating factors. We obtain the center conditions when the system (1) has an irreducible invariant conic $a_{20}x^2+a_{11}xy+a_{02}y^2+a_{10}x+a_{01}y+1=0$, where $a_{20}, a_{11}, a_{02}, a_{10}, a_{01}$ are real parameters.

Bibliography

[1] D. Cozma, *Integrability of cubic systems with invariant straight lines and invariant conics.* Ştiinţa, Chişinău, 2013.

[2] D. Cozma, *Darboux integrability of a cubic differential system with two parallel invariant straight lines.* Carpathian J. Math., 2022, **38** (1), 129–137.

[3] J. Llibre, *On the centers of cubic polynomial differential systems with four invariant straight lines.* Topological Methods in Nonlinear Analysis, 2020, **55** (2), 387–402.

[4] D. Cozma, A. Dascalescu, *Integrability conditions for a class of cubic differential systems with a bundle of two invariant straight lines and one invariant cubic.* Buletinul Academiei de Ştiinţe a Republicii Moldova. Matematica, 2018, **86** (1), 120–138.

[5] D. Cozma, A. Matei, *Integrating factors for a cubic differential system with two algebraic solutions.* ROMAI Journal, 2021, **17** (1), 65–86.

[6] D. Cozma, *Center conditions for a cubic differential system having an integrating factor.* Bukovinian Mathematical Journal, 2022, **10** (1), 22–32.

On the stability of a time-invariant linear system with time-delay and disturbances with application to aerospace engineering

Daniela Enciu, Ioan Ursu, Adrian Toader

INCAS - National Institute for Aerospace Research "Elie Carafoli" Bd. Iuliu Maniu 220, 061126 Bucharest, Romania
e-mail: daniella_enciu@yahoo.com, ursu.ioan@incas.ro

The beauty of the mathematical equations is highlighted by presenting a practical, real application in controlling the flight stability of an airplane in a turbulent atmospheric field. This paper presents a linear mathematical model with actuator delay in the control chain and external disturbances. The time-delay is compensated by applying a state predictive feedback method and the perturbation is treated according to the basic Kolmogorov concept.

Acknowledgment: *This work was supported by grants of the Romanian Ministry of Research, Innovation and Digitization, CNCS – UEFISCDI, project number PN-III-P1-1.1-PD-2021-0610, PN-III-P2- 2.1-PED-2021-2265, PN-III-P2-2.1-PED-2021-4147, and by NUCLEU Programme project code PN 23-17-07-01, Ctr. 36 N/12.01.2023.*

On the Dynamics of Stochastic Systems and Fluctuation-Dissipation Theorem

Raluca Georgescu[1] and Eugen Vasile[2]

[1]*National University of Science and Technology Politehnica Bucharest - Pitesti University Center,*
[2]*National University of Science and Technology Politehnica Bucharest*
e-mail: [1]raluca.georgescu@upit.ro, [2]evasevas@yahoo.com

A signal with fluctuating excitation is described by equations with random terms. A stochastic dynamical system is subjected to the effects of random noise. In the general framework of these systems, various formulations of the fluctuation–dissipation theorem and possibilities for describing noise in electrical systems and devices are analyzed. Some examples related to brownian motion, thermal noise, light absorption are discussed. Linear Response Theory is also considered.

Keywords: dynamical system, stochastic, fluctuation-dissipation theorem, noise

The absolute stabilization and optimal control of hydrofoil watercraft

Mircea Lupu

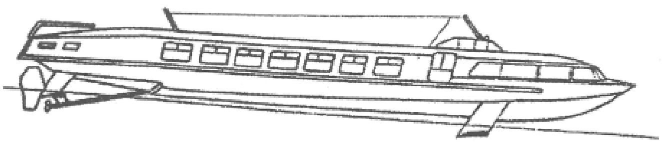
"Transilvania" University of Brasov
Academy of Romanian Scientst, Romania
e-mail: emlupu2006@yahoo,com

The hydrofoil longitudinal movement equation(without rolling or gyration disturbance) in the speed space $v(t)$ is:

$$\frac{\mathrm{d}v}{\mathrm{d}t} = T(v) - S(v)$$

where $T(v)$ is the traction force(motor or turbo-jet) and $S(v)$ is the drag force resultant for the unity mass ship. Generally, $T(v)$ is linear decreasing with respect to speed v and $S(v)$ is a monotone function on intervals. .

The paper deals with an analytcally and numerically study of the stable movement regime in the vicinity of the critical (equilibrium) points v_1, v_2, v_3. The asymptotical stability is highlighted by using the Liapunov criteria too.



The practical control (self-stabilization) is obtained by using an automate speed controller (scanner) depending on the wings flaps and the ship attack angle. Transversal waves or wind disturbances causes a ship flutter effect. For hydrofoil dynamics, an optimal control is applied finally, using the minimal time criterion related to Pontryagin's maximum principle.

On the Jacobi Stability of a Chaotic System

Florian Munteanu

Department of Applied Mathematics, Faculty of Sciences, University of Craiova, Romania
e-mail: `florian.munteanu@edu.ucv.ro`

In this work, a quadratic four dimensional smooth chaotic system with multiple coexisting attractors is considered. By reformulating the first-order differential system as a system with four second-order differential equations, we will investigate the nonlinear dynamics of the system from the Jacobi stability perspective through the Kosambi-Cartan-Chern (KCC) geometric theory. The intrinsic geometric properties of the systems will be studied by determining the associated geometric objects: the zero-connection curvature tensor, the nonlinear connection, the Berwald connection, and the five KCC invariants (the external force - the first invariant; the deviation curvature tensor - the second invariant; the torsion tensor - the third invariant; the Riemann–Christoffel curvature tensor - the fourth invariant; the Douglas tensor - the fifth invariant). In order to obtain necessary and sufficient conditions for the Jacobi stability near each equilibrium point, the deviation curvature tensor will be determined at each equilibrium point. Furthermore, in order to deepen our knowledge about the dynamics of this chaotic dynamical system, we will compare the Jacobi stability with the classical Lyapunov (linear) stability.

About computing of ordinary Hilbert series for Sibirsky graded algebras of differential system $s(3,7)$

Pricop Victor[1], Tacu Mariana[2]

[1] *Technical University of Moldova,*
[2] *Academy of Economic Studies of Moldova*
Chişinău, Republic of Moldova

e-mail: `victor.pricop@mate.utm.md, tacu.mariana@ase.md`

Consider a differential system $s(\Gamma)$ of the form

$$\dot{x}^j = \sum_{i=0}^{\ell} P^j_{m_i}(x), \ \ (j = 1, 2; \ \ell < \infty), \tag{1}$$

where $\Gamma = \{m_i\}_{i=0}^{\ell}$ is a finite set of distinct nonnegative integers. One of research methods of system $s(\Gamma)$ is the method of algebraic invariants [1] and Lie algebras [2]. For the study of centro-affine invariants, which in relation to the unimodular group form a finitely determined graded algebras of comitants S_Γ and invariants SI_Γ [2], called Sibirsky algebras [3], it is necessary to use Hilbert series of these algebras, which were constructed for $\Gamma \subset \{0, 1, 2, 3, 4, 5\}$ using the generalized Sylvester method. But for more complicated Γ, this method encounters insurmountable computational difficulties.

In the paper [4] is described the method of computing Hilbert series of invariants ring using Residue Theorem. This method was adapted to computing an ordinary Hilbert series for Sibirsky graded algebras of comitants and invariants of differential systems [3] by using corresponding generating functions [2]. It was obtained an ordinary Hilbert series for Sibirsky graded algebras of comitants $H_{S_{3,7}}$ and invariants $H_{SI_{3,7}}$ of the differential system (1) with $\Gamma = \{3, 7\}$.

Lemma 1. *For the differential system $s(3,7)$ the following ordinary Hilbert series for Sibirsky graded algebras of comitants and invariants was obtained*

$$H_{S_{3,7}}(u) = \frac{1}{(1-u)^3(1+u)^7(1-u^4)^4(1-c^3)^7(1-u^5)^5(1-u^9)(1-u^7)^3}(1+$$
$$+4u + 7u^2 + 12u^3 + 63u^4 + 324u^5 + 1298u^6 + 4413u^7 + 13568u^8 + 38708u^9+$$
$$+103376u^{10} + 259115u^{11} + 611655u^{12} + 1365235u^{13} + 2893063u^{14}+$$
$$+5841334u^{15} + 11272533u^{16} + 20846552u^{17} + 37030793u^{18} + 63314999u^{19}+$$
$$+104392569u^{20} + 166252905u^{21} + 256123279u^{22} + 382194740u^{23}+$$
$$+553088300u^{24} + 777041306u^{25} + 1060844290u^{26} + 1408607557u^{27}+$$
$$+1820502258u^{28} + 2291653983u^{29} + 2811400597u^{30} + 3363096026u^{31}+$$
$$+3924600268u^{32} + 4469498866u^{33} + 4969004038u^{34} + 5394358510u^{35}+$$

$$+5719487780u^{36}+5923562227u^{37}+5993136400u^{38}+5923562227u^{39}+$$
$$+5719487780u^{40}+5394358510u^{41}+4969004038u^{42}+4469498866u^{43}+$$
$$+3924600268u^{44}+3363096026u^{45}+2811400597u^{46}+2291653983u^{47}+$$
$$+1820502258u^{48}+1408607557u^{49}+1060844290u^{50}+777041306u^{51}+$$
$$+553088300u^{52}+382194740u^{53}+256123279u^{54}+166252905u^{55}+$$
$$+104392569u^{56}+63314999u^{57}+37030793u^{58}+20846552u^{59}+11272533u^{60}+$$
$$+5841334u^{61}+2893063u^{62}+1365235u^{63}+611655u^{64}+259115u^{65}+$$
$$+103376u^{66}+38708u^{67}+13568u^{68}+4413u^{69}+1298u^{70}+324u^{71}+63u^{72}+$$
$$+12u^{73}+7u^{74}+4u^{75}+u^{76}),$$

$$H_{SI_{3,7}}(z)=\frac{1}{(1-c^2)^3(1+z)^3(1-z^3)^7(1-z^5)^4(1-z^7)^2(1-z^4)^5}(1+3z+$$
$$+4z^2+7z^3+48z^4+231z^5+849z^6+2634z^7+7449z^8+19573z^9+$$
$$+48098z^{10}+110393z^{11}+237525z^{12}+480950z^{13}+920782z^{14}+1672880z^{15}+$$
$$+2893570z^{16}+4776482z^{17}+7541247z^{18}+11408625z^{19}+16566119z^{20}+$$
$$+23122522z^{21}+31063051z^{22}+40208147z^{23}+50194730z^{24}+60480527z^{25}+$$
$$+70384949z^{26}+79154876z^{27}+86057490z^{28}+90474840z^{29}+91995796z^{30}+$$
$$+90474840z^{31}+86057490z^{32}+79154876z^{33}+70384949z^{34}+60480527z^{35}+$$
$$+50194730z^{36}+40208147z^{37}+31063051z^{38}+23122522z^{39}+16566119z^{40}+$$
$$+11408625z^{41}+7541247z^{42}+4776482z^{43}+2893570z^{44}+1672880z^{45}+$$
$$+920782z^{46}+480950z^{47}+237525z^{48}+110393z^{49}+48098z^{50}+19573z^{51}+$$
$$+7449z^{52}+2634z^{53}+849z^{54}+231z^{55}+48z^{56}+7z^{57}+4z^{58}+3z^{59}+z^{60}).$$

According to [2,5] the Krull dimension for finitely determined algebras is equal to the order of pole of corresponding ordinary Hilbert series at the unit, then using Lemma 1 we obtain that there takes place.

Theorem 1. *The Krull dimensions of Sibirsky graded algebras* $S_{3,7}$ ($SI_{3,7}$) *is equal to* 23 (21)*, respectively.*

Remark 1. *The Krull dimension gives us the maximal number of algebraically independent comitants and invariants of Sibirsky graded algebras* $S_{3,7}$ *and* $SI_{3,7}$ *of differential system* $s(3,7)$.

Bibliography

[1] Sibirsky K. S. *Introduction to the algebraic theory of invariants of differential equations.* Manchester-New York, Manchester University Press 1988. VII, 169 p.

[2] Popa M. N. *Algebraic methods for differential systems.* Editura the Flower Power, Universitatea din Piteşti, Seria Matematica Aplicată şi Industrială, 2004, (15) (in Romanian).

[3] M. N. Popa, V. V. Pricop. *The Center and Focus Problem: Algebraic Solutions and Hypotheses.* Published September 24, 2021 by Chapman and Hall/CRC, 226 p.

[4] Harm Derksen, Gregor Kemper. *Computational Invariant Theory.* Encyclopaedia of Mathematical Sciences, vol. 130, Springer-Verlag, Berlin, 2002, 268 p.

[5] Springer T. A. *Invariant Theory.* New in foreign science, t. 24, Mathematics, Moscow, Mir, 1981, 191p. (in Russian).

A qualitative study of the quadratic differential systems with the line at infinity of maximal multiplicity

Vadim Repeşco

"Ion Creangă" State Pedagogical University of Chişinău, Republic of Moldova
e-mail: `repescov@gmail.com`

Consider a quadratic differential system featuring a singular point characterized by purely imaginary eigenvalues, signifying a center-focus singularity. The work in reference [1] establishes that the highest multiplicity of the invariant straight line at infinity for this system is three. In this exposition, we employ various methodologies to elucidate the qualitative dynamics of these systems on the Poincaré disk. Furthermore, we intend to employ these findings to perform a comparative analysis with previously obtained similar results.

Stability and bifurcations in a model from ecology

Mihaela Sterpu, Raluca Efrem, and Carmen Rocşoreanu

University of Craiova, Romania
e-mail: `mihaela.sterpu@edu.ucv.ro, raluca.efrem@edu.ucv.ro, carmen.rocsoreanu@edu.ucv.ro`

A 4D model for a closed ecosystem with four compartments is derived. The model is reduced to a 3D dynamical system, described by a system of three nonlinear ordinary differential equations, depending on seven positive parameters. The local dynamics and bifurcations of this model are investigated. The system possesses at most three equilibrium points. It is found that at most one of the equilibria is locally asymptotically stable for each parameter strata. Several codimension one bifurcations are determined and analyzed. Various attractors and transitions scenarios are emphasized numerically.

Bibliography

[1] M. Kloosterman, S.A. Campbell, F.J. Poulin, *A closed NPZ model with delayed nutrient recycling*, J. Math. Biol. **68** (2014), 815–850.

[2] L. Perko, *Differential Equations and Dynamical Systems* (Third Edition), Springer Verlag, New York, 2001.

[3] M. Sterpu, C. Rocşoreanu, R. Efrem, S.A. Campbell, *Stability and bifurcations in a Nutrient–Phytoplankton–Zooplankton model with delayed nutrient recycling with Gamma distribution*, Mathematics **11** (2023), no. 13, Art. 2911. https://doi.org/10.3390/math11132911

Quadratic differential systems with the line at infinity of maximal multiplicity

Alexandru Şubă

"Vladimir Andrunachievici" Institute of Mathematics and Computer Science,
"Ion Creangă" State Pedagogical University,
Chişinău, Republic of Moldova
e-mail: alexandru.suba@math.md

I. Systems of the general form. We consider the real quadratic system of differential equations

$$\begin{cases} \dot{x} = p_0 + p_1(x,y) + p_2(x,y) \equiv p(x,y), \\ \dot{y} = q_0 + q_1(x,y) + q_2(x,y) \equiv q(x,y), \ \gcd(p,q) = 1, \ yp_2 - xq_2 \not\equiv 0, \end{cases} \tag{1}$$

where p_j, q_j are polynomial in x and y of degree j.

The homogeneous system associated with the system (1) has the form

$$\begin{cases} \dot{x} = p_0 Z^2 + p_1(x,y)Z + p_2(x,y) \equiv P(x,y,Z), \\ \dot{y} = q_0 Z^2 + q_1(x,y)Z + q_2(x,y) \equiv Q(x,y,Z). \end{cases}$$

Denote $\mathbb{X} = P(x,y,Z)\frac{\partial}{\partial x} + Q(x,y,Z)\frac{\partial}{\partial y}$.

We say that the line at infinity $Z = 0$ has *multiplicity* $m+1$ if m is the greatest positive integer such that Z^m divides $E_\infty = P \cdot \mathbb{X}(Q) - Q \cdot \mathbb{X}(P)$.

Theorem 1. *In the class of quadratic differential systems* (1) *the maximal multiplicity of the line at infinity is five. Modulo affine transformations and time rescaling, each quadratic system with the line at infinity of multiplicity five has the form* $\dot{x} = 1, \ \dot{y} = b_0 + b_1 x + x^2, \ b_0, b_1 \in \mathbb{R}$.

II. Centers. If (1) has a critical point with purely imaginary eigenvalues, then using affine transformations and time rescaling, it can be written in the form

$$\begin{cases} \dot{x} = y + ax^2 + cxy + fy^2 \equiv p(x,y), \\ \dot{y} = -(x + gx^2 + dxy + by^2) \equiv q(x,y), \\ \gcd(p,q) = 1, gx^3 + (a+d)x^2y + (b+c)xy^2 + fy^3 \not\equiv 0. \end{cases} \tag{2}$$

Theorem 2. *In the class of quadratic differential systems* (2)*, the maximal multiplicity of the line at infinity is three. Any quadratic system which has the line at infinity of multiplicity 3 has one of the following three forms:*

$$\dot{x} = (1+cx)y, \ \dot{y} = -x, \ c \neq 0; \tag{3}$$

$$\dot{x} = y, \ \dot{y} = -(1+dy)x, \ d \neq 0; \tag{4}$$

$$\begin{cases} \dot{x} = y - f(x^2 - y^2) + (f^2 - g^2)xy/g, \\ \dot{y} = -(x + g(x^2 - y^2) + (g^2 - f^2)xy/f). \end{cases} \tag{5}$$

Remark 1. 1) *The transformation* $x \to y, \ y \to x, \ d \to c, \ t \to -t$ *reduce the system* (4) *to the system* (3);

2) *The system* (3) *(respectively,* (4), (5)*) has the affine invariant straight line* $1 + cx = 0$ *(respectively,* $1 + dx = 0, \ fg + f(f^2+g^2)x + g(f^2+g^2)y = 0$*).*

Theorem 3. *Let the quadratic differential system* (1) *satisfy the following conditions: a) it has a critical point* M_0 *with purely imaginary eigenvalues, and b) the line at infinity for* (1) *is of maximal multiplicity. Then,*

– M_0 *is a center;*
– (1) *has an affine invariant straight line* $\mathcal{L} = 0$*;*
– $\mu(x,y) = 1/\mathcal{L}$ *is an integrating factor for* (1).

3. Mathematical Modeling

Investigation of interaction of noble metals (Cu, Ag, Au, Pt and Ir) with Nano-sheets

Mansoor H. Alshehri

Mathematics Department, College of Science, King Saud University, P.O. Box 2455, Riyadh 11451, Saudi Arabia
e-mail: mhalshehri@ksu.edu.sa

Two-dimensional nano-materials such as graphene and hexagonal boron nitride nano-sheets Fig.1 have attracted tremendous interest in the research community and starting-point for the development of nanotechnology. Using classical applied mathematical modeling, we derive explicit analytical expressions to determine the binding energies of noble metals including copper, silver, gold, platinum and Iridium (Cu, Ag, Au, Pt and Ir) atoms on graphene and hexagonal boron nitride nano-sheets, as shown in Fig.2 . We adopt the 6–12 Lennard-Jones potential function together with the continuous approach to determine the preferred minimum energy position of an offset metal atom above the surface of the graphene and hexagonal boron nitride nano-sheets.

The main results of this study is an analytical expressions for the interaction energies that we then utilize to report the mechanism of adsorption of the metal atoms on graphene and hexagonal boron nitride surfaces. Results observe that the minimum binding energy is occured when Cu, Ag, Au, Pt and Ir are is set at a perpendicular distances in the region from 3.302 Å to 3.683 Å above the nano-sheet surface which correspond to an adsorption an adsorption energies in the region from 0.842 to 2.978 (kcal/mol). Our results might assist to provide the main information on the interaction energies between the metal atoms and the two-dimensional nano-materials.

Keywords: noble metals, graphene, h-BN, mathematical modeling, Lennard–Jones potential.

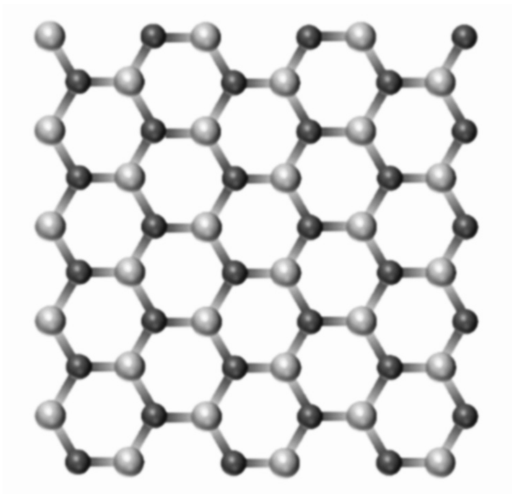


Fig. 1. Two-dimensional sheet.

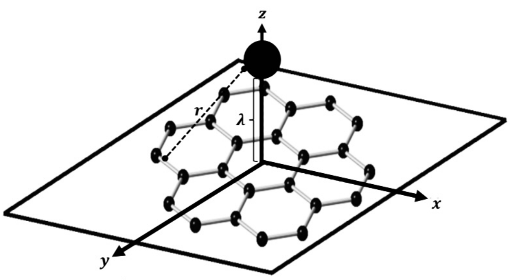


Fig. 2. A sketch of the interaction between metal atoms and nano-sheet.

Mathematical Models for Object Detection and Tracking

Tudor Barbu

Institute of Computer Science of the Romanian Academy - Iasi Branch, ROMANIA
e-mail: tudor.barbu@iit.academiaromana-is.ro

This research work address the moving object detection and tracking domain, describing some mathematical models that have been appplied successfully in this computer vision field. Thus, several variational and non-variational partial differential equation (PDE)-based models for image and video object detection and tracking are surveyed here.

The detection and tracking techniques based on Geometric Active Contour models, also called snakes, which represent energy-based (variational) image segmentation schemes, are presented first. Then, another category of PDE-based geometric models for object detection and tracking, containing mathematical models based on level-set functions, is disscused here.

Moving object tracking approaches based on the video optical flow that is estimated using PDE-based models are described next. The histogram-based PDE models for video object tracking are then presented. Image object detection techniques using PDE-based boundary and contour extraction models are also discussed.

Finally, our own contributions in this research domain are described here. Thus, some nonlinear PDE-based automatic detection and tracking frameworks for certain classes of video objects, such as pedestrians and vehicles, are briefly presented.

Bibliography

[1] V. Caselles, R. Kimmel, G. Sapiro, *Geodesic active contours*, International Journal of Computer Vision 22 (1) (1997), 61-79.

[2] T. F. Chan, L. A. Vese, *Active contours without edges*, IEEE Transactions on Image Processing, 10 (2) (2001) 266-277.

[3] T. Barbu, *Robust contour tracking model using a variational level-set algorithm*, Numerical Functional Analysis and Optimization, 35 (3) (2014) 263-274.

[4] M. Chihaoui, A. Elkefi, W. Bellil, C. Amar, *Detection and tracking of the moving objects in a video sequence by geodesic active contour*, 13^{th} International Conference on Computer Graphics, Imaging and Visualization, CGiV March 2016. IEEE, 212-215.

[5] T. Barbu, *Novel Approach for Moving Human Detection and Tracking in Static Camera Video Sequences*, Proc. of the Romanian Academy, Series A, 13 (3) (2012), 269-277.

[6] K. Horn, G. Schunck, *Determining optical flow*, Artificial Intelligence 17 (1–3) (1981), 185—203.

[7] P. Li, L. Xiao, *Histogram-based partial differential equation for object tracking*, 7^{th} International Conference on Advances in Pattern Recognition, 2009, 286-289.IEEE

[8] N. Paragios, R. Deriche, *Geodesic active contours and level sets for the detection and tracking of moving objects*, IEEE Transactions on pattern analysis and machine intelligence 22 (3) (2000), 266-280.

[9] M. Yokoyama, T. Poggio, *A contour-based moving object detection and tracking*, 2006 IEEE international workshop on visual surveillance and performance evaluation of tracking and surveillance, 2005, 271-276.

[10] T. Barbu, *Novel Diffusion-Based Models for Image Restoration and Interpolation*, Berlin/Heidelberg, Germany, Springer International Publishing, 2019.

[11] Y. Yuan, H. Yang, Y. Fang, W. Lin, *Visual object tracking by structure complexity coefficients*, IEEE Transactions on Multimedia, 17 (8) (2015), 1125-1136.

[12] K. Kale, S. Pawar, P. Dhulekar, *Moving object tracking using optical flow and motion vector estimation*, 4^{th} International conference on reliability, infocom technologies and optimization (ICRITO), Sept. 2015, 1-6. IEEE

[13] T. Barbu, *An Automatic Unsupervised Pattern Recognition Approach*, Proceedings of the Romanian Academy, Series A, 7(1), January-April (2006), 73-78.

[14] T. Barbu, *Deep Learning-based Multiple Moving Vehicle Detection and Tracking using a Nonlinear Fourth-order Reaction-Diffusion based Multi-scale Video Object Analysis*, Discrete & Continuous Dynamical Systems - Series S, 16 (1), 2023, 6—32.

[15] T. Barbu, *Multiple Pedestrian Tracking Framework using Deep Learning-based Multiscale Image Analysis for Stationary-camera Video Surveillance*, 8^{th} IEEE International Smart Cities Conference 2022, ISC2 2022, Paphos, Cyprus, 26-29 sept. 2022.

Some solutions for applications of metric spaces in medicine

Mariana Butnaru, Gheorghe Capatana, Alexandru Popov

Moldova State University, Republic of Moldova
e-mail: mariana.butnaru@usm.md, gheorghe.capatana@usm.md, san.popov90@gmail.com

The authors of the paper study with artificial intelligence methods the mechanisms of mental and behavioral disorders in epilepsy (MBDE) remissions.

Scientific researches has gone through the following stages.

Description of MBDE research field. A book was developed [1] for description of the MBDE research field. RF MBDE operates with 27 MBDE diagnoses and a control diagnosis G40 (epilepsy without MBDE), 17 syndromes and 163 symptoms.

Development of MBDE RF knowledge base. BC TPCE consists of 19 compartments and uses numerical and symbolic fuzzy values Knowledge base.

Development of 19 metric spaces for evaluating distances between MBDE RF diagnostics.

Development of 19 metric spaces for evaluating the similarities of MBDE RF diagnoses.

An album of 76 MBDE distance and similarity evaluation tables was developed.

The results of scientific research have been recognized with diplomas of excellence and gold medals at some international innovation and scientific research exhibitions and book fairs.

Bibliography

[1] Popov Al., Butnaru M., Capatana Gh., Capatana A. *Mental and behavioral disorders in epilepsy: classification, diagnosis, behavioral algorithms, anamnesis, clinical manifestations, paraclinical investigations, treatment, rehabilitation, necessary resources, prophylaxis.* Chisinau: Moldova State University, 2018, 118 p., sourse: http://dspace.usm.md:8080/xmlui/handle/123456789/2013 (in Romanian)

Closed-loop model estimation of the underdamped second order inertial systems

Irina Cojuhari

Technical University of Moldova, Faculty of Computers, Informatics and Microelectronics, Department of Software Engineering and Automatics, Chişinău, Republic of Moldova

e-mail: `irina.cojuhari@ati.utm.md`

In this paper is proposed an algorithm for model estimation of the underdamped second order inertial systems in the closed-loop, where the system is proposed to be approximated with the following transfer function:

$$H_F = \frac{k\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{k}{a_0 s^2 + a_1 s + a_2},$$

where ω_n is natural frequency, ξ - damping ratio, k - transfer coefficient and $a_0 = \frac{1}{\omega_n^2}$, $a_1 = \frac{2\xi}{\omega_n}$, $a_2 = 1$.

The block scheme of the closed-loop system is presented in the Figure 1.

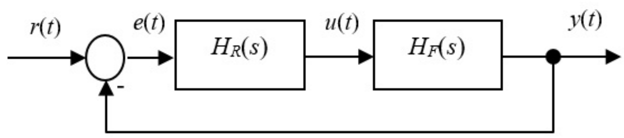


Figure 1. Structural scheme of the automatic control system.

For model estimation is proposed to be used P controller with transfer function:

$$H_R(s) = k_p,$$

where k_p is proportional tuning parameter.

The algorithm for closed-loop model identification is the following:

1. Implementation of the feedback control system with P controller.
2. Variation of the proportional tuning parameter $k_p > 0$ until the system achieves underdamped step response, as is presented in the Figure 2.
3. From obtained underdamped step response Figure 2 calculation the period of oscillations - T_0.
4. From Figure 2 calculation the value of the damping ratio [1-2]:

$$\xi = \frac{1}{\sqrt{1 + (\frac{2\pi}{\log d})^2}},$$

where d is decay ratio.

5. Calculation of the system transfer coefficient according to the relation:

$$k = \lim_{n\to\infty} \frac{\Delta y}{\Delta u} = \lim_{n\to\infty} \frac{y_{st} - y_{init}}{u - u_{init}},$$

where y_{st} is the steady-state output value, $y_{initial}$ is the initial value of the output response, u – input signal, u_{inital} is the initial value of the input signal.

6. Calculation the value of natural frequency according to the relation [1-2]:

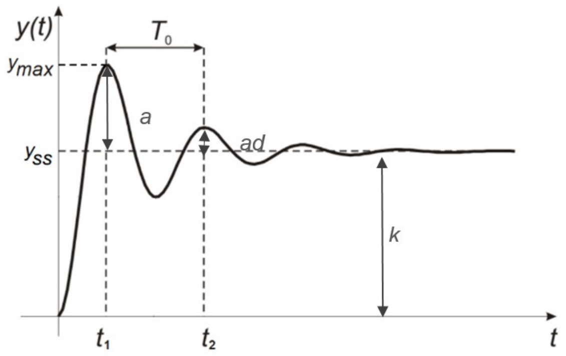


Figure 2. Underdamped step response of the closed-loop system.

$$\omega_n = \frac{2\pi}{T_0\sqrt{1-\xi^2}}.$$

7. Calculation of the $H_F(s)$ transfer function parameters:

$$a_0 = \frac{k_{cr}k+1}{\omega_n^2},\ a_1 = \frac{2(k_{cr}k+1)\xi}{\omega_n},\ a_2 = 1.$$

Acknowledgment. *This work was supported by the project 22.00208.5007.12/PD I "Models and algorithms for control and behaviour approach of the interoperable systems" and by the project 20.80009.5007.26 „Models, algorithms and technologies for the control, optimization and security of the Cyber-Physical systems".*

Bibliography

[1] Dumitrache I. (coord.) *Automatica.* Editura Academiei Române, vol. II, 2009.

[2] Cojuhari I. *Algorithm for Self-Tuning the PID Controller.* In Journal of Engineering Scienc. 2021, Vol. XXVIII (4), pp. 63-73.

Flow Dynamics on Curved Surfaces

Stefan-Gicu Cruceanu, Dorin Marinscu

"Gheorghe Mihoc-Caius Iacob" Institute of Mathematical Statistics and Applied Mathematics of Roamnian Academy

e-mail: `stefan.cruceanu@ismma.ro, marinescu.dorin@ismma.ro`

The effects of curvature on the flow dynamics for the simplified and widely used Shallow Water Equation models are only caught through the gradient of the base flow surface. We are trying here to understand if extending the model by explicitly including terms related to the curvature of the surface is necessary to increase the accuracy of the phenomena. A comparison of the solutions of two models for the case of steady flow on radial symmetric surface will help us draw some conclusions.

Global stability of coexistence equilibria for n-species models of facultative mutualism

Paul Georgescu

"Gheorghe Asachi" Technical University of Iaşi, Romania
e-mail: `v.p.georgescu@gmail.com`

We further pursue our earlier investigations on a framework for analyzing the dynamics of n-species models of facultative mutualism, providing general sufficient conditions for the global stability of the positive equilibrium. These conditions, consisting of inequalities involving reproductive ratios that are computed at high population densities, augmented by either sublinearity and monotonicity properties, are then observed to be valid for several n-species versions of commonly used models of mutualism, yielding stability criteria with practical relevance.

Applications of the Laplace Transform to chemical reactor design and the role of inverse problems

Victor Martinez-Luaces

University of the Republic of Uruguay, University of Granada, Spain.
e-mail: `victorml@correo.ugr.es`

In this paper, different reactor systems are studied, being all of them combinations of ideally stirred continuous tank reactors (CSTR), whose transfer functions - quotient of Laplace transforms of the reactor output and input - are always rational functions.
The inverse problem consists of obtaining the reactor or system of reactors that corresponds to a certain given rational function, assuming that it exists. Therefore, the mentioned inverse problem gives rise to the study of conditions of existence, uniqueness, and stability of the solutions.
In this work, this inverse problem is analyzed and it is concluded that the solution does not always exist, if it exists, it is not necessarily unique and, even more, small changes in the given rational function can produce very important changes in the reactor obtained and so, stability does not take place as it usually occurs in many typical direct problems.
Several examples are presented in the article illustrating all the above results.

Keywords: Laplace transform, chemical reactors design, inverse problems.

A variant of saddle point theorem with applications

Irina Meghea

National University of Science and Technology POLITEHNICA Bucharest Faculty of Applied Sciences, Bucharest
e-mail: i_meghea@yahoo.com

A generalized variant of saddle point theorem (SPT) involving the β-differentiability is used in obtaining and characterizing weak solution for a special class of mathematical physics equations with p-Laplacian and p-pseudo-Laplacian. These results are used to solve some Dirichlet and von Neumann type problems evolved from modeling real phenomena. The novelty of this work consists in the application of that special kind of SPT to obtain statements for the cited problems together with their usage to draw solutions for real models.

Validity conditions for the quasi-steady-state approximation of an enzyme kinetics model with inhibition and substrate input

A.-M. Moşneagu[1,2,3,4] and I. Stoleriu[1,5]

[1] *"Alexandru Ioan Cuza" University of Iaşi, Faculty of Mathematics,Iaşi, Romania;*
[2] *"Alexandru Ioan Cuza" University of Iaşi, Research Center with Integrated Techniques for Atmospheric Aerosol Investigation in Romania, RECENT AIR,*
[3] *Laboratory of astronomy and astrophysics,Iasi, Romania*
e-mail: [4]anamaria.mosneagu@uaic.ro, [5]iulian.stoleriu@uaic.ro

In enzyme kinetics, the enzyme-catalysed reactions are some biochemical reactions in which enzymes speed up the conversion of substrate into product by lowering their activation energy. In these reactions, some of the chemical species exhibit different intrinsic time scales and tend to reach an equilibrium state much quicker than the other species. A commonly used mechanism to simplify the description of the dynamics of such biological systems is the quasi-steady-state approximation. In this paper, we investigate the validity of the standard quasi-steady-state approximation for the Michaelis-Menten reaction model with inhibition and a constant substrate input. Necessary and sufficient conditions for the validity of these assumptions were derived and were shown to be dependent, among others, on the substrate input. The validity conditions are verified numerically using the classical Runge-Kutta method.

A Comparative Analysis of L^1, L^2, and L^∞ norms for Image Segmentation Algorithms: Sobel, Prewitt, and Canny

Silviu-Dumitru Pavăl

"Gheorghe Asachi" Technical University of Iasi,
Faculty of Automatic Control and Computer Engineering,
Department of Computer Science and Engineering
e-mail: silviu-dumitru.paval@academic.tuiasi.ro

Image segmentation is a fundamental task in computer vision with ap- plications ranging from medical imaging to object detection. Evaluating the performance of segmentation algorithms is crucial for selecting the most appropriate method for specific applications. This article presents a comprehensive comparative analysis of three popular edge detection al- gorithms—Sobel, Prewitt, and Canny—using three distinct distance met- rics: L^1, L^2, and L^∞. The study assesses the effectiveness of each algorithm-metric combination in capturing edge information accurately and efficiently.

Through experimentation on diverse datasets, we quantitatively eval- uate the precision, robustness, and computational efficiency of the Sobel, Prewitt, and Canny algorithms under different metric norms. The L^1 norm, representing the Manhattan distance, emphasizes the ability of algorithms to capture edges in a more localized manner. The L^2 norm, measuring the Euclidean distance, emphasizes global consistency in edge detection. The L^∞ norm, based on the maximum norm, highlights the algorithms' ability to detect prominent edges in the presence of noise.

Our findings reveal insights into the strengths and weaknesses of each algorithm-metric combination, shedding light on their performance trade- offs in various scenarios. This study contributes to a better understanding of the interplay between edge detection algorithms and distance metrics, aiding practitioners in making informed decisions when selecting appropri- ate techniques for image segmentation tasks. Additionally, it underscores the significance of considering multiple evaluation metrics to obtain a comprehensive assessment of algorithm performance.

Multiscale models for particle-laden flow through periodic networks

Arcady Wey

Mathematical Institute, University of Oxford, United Kingdom
e-mail: arkady.wey@maths.ox.ac.uk

Experimental observations are challenging. In filtration, it is difficult to analyse particle deposition without destroying the membrane, and there are many ethical considerations involved with brain observation. Researchers use mathematical modelling for insight. Finite element models (FEMs), involving fine reconstruction of the materials' microscale, are one typical approach. Although these are usually highly accurate, they are often prohibitively computationally expensive for tasks that involve repeated simulation, such as parameter-regime optimisations.

Relevant materials can be well represented by periodic networks of edges, which correspond to conductive channels, and nodes, which correspond to junctions between them. Recent advances in visualisation technologies mean networks with the correct structure can be algorithmically extracted from high-quality images of materials' microscale. This development opens up the potential use of network models (NMs), which are 'dynamical systems' of algebraic–differential equations on each node and edge. Unfortunately, material networks often consist of billions of nodes and edges – porous membranes can contain many fibrils, and there are typically billions of neurons in a human brain. Thus, the resulting algebraic problem to be solved is large and so NMs suffer from speed and memory problems, even when they are linear.

In this talk, we develop an asymptotic method to homogenise (systematically average) NMs to obtain network–continuum multiscale models (MMs). We refer to this method as 'network homogenisation', a discrete version of the usual continuous method of multiple scales. To exemplify, we write down a simple NM representing the advection and deposition of particles in a network, and the resulting changes to channel conductances. The result of network homogenisation is a system of continuous partial differential equations (PDEs) that models macroscale behaviour. In our case, this consists of Darcy's equation for the flow coupled to an advection–reaction equation for the particle concentration. Unlike other PDE models, microscale information enters this macroscale system via its parameters. We call these the permeability and adhesivity of the network, since they measure its effective conductance and adherence. They depend on channel conductances, which are given by solution of a linear algebraic problem that is significantly smaller than that arising from the original NM.

In this way, MMs arise as an alternative to FEMs and NMs. MMs approach the accuracy of their associated NMs, but can be much more computationally feasible, opening up their potential use in optimisation routines.

Modeling of the adsorption of methylene blue onto clayey carbonate diatomite

Veaceslav Zelentsov[1], Oleg Bolotin[2], Tatiana Datsko[1]

[1] *The Institute of Applied Physics,*
[2] *The Institute of Geology and Seismology, State University of Moldova, Chişinau*
e-mail: vzelen@yandex.ru

The application of kinetic models of pseudo- first, pseudo- second order and Elovich model for the description of experimental kinetic data related to the adsorption of methylene blue (MB) on the clayey carbonate diatomite (CCD) is discussed in this work. The model pseudo-first order of Lagergren [1] was used in form

$$\ln(a_m - a_t) = \ln a_m - k_1 t. \qquad (1)$$

The pseudo-second order Mackay and Ho model [2] is expressed as

$$\frac{t}{a_t} = \frac{1}{k_2 * a_m^2} + \frac{t}{a_m}. \qquad (2)$$

The model of Elovich [3] is expressed as

$$a_t = \frac{1}{\beta}\ln(\alpha\beta) + \frac{1}{\beta}\ln t. \qquad (3)$$

In the models a_m and a_t [$\mathrm{mg\,g^{-1}}$] stand for the amounts of MB adsorbed at equilibrium and at time t[min], respectively; k_1[$\mathrm{min^{-1}}$] is the pseudo-first-order rate constant, k_2[$\mathrm{g\,mg^{-1}\,min^{-1}}$ is the rate constant of the pseudo-second order kinetics and α and β are constants of equation of Elovich.

Employing the linearized form of pseudo-first-order model $\ln(a_m - a_t) = f(t)$, Eq. (1)) and linearized form of pseudo-second-order model t/a_t , versus contact time t, Eq. (2) and a_t vs $\ln t$ for the Elovich model (3) we determined the constants k_1, k_2, α, β anda_m values using the slopes and intercept points of the linear plots.

Table 1. The kinetic parameters of adsorption of MB onto CCD and correlation coefficients R2 of kinetics models

Kinetic Model	Parameter	a_m^{calc}	a_m^{exp}	R^2
Pseudo 1	$k_1 = 0.0543$	20.124	89.9	0.4630
Pseudo 2	$k_2 = 1.151$	86.90	89.9	0.9924
Pseudo 3	$\beta = 0.329$ $\ln(\alpha\beta) = 24.728$	86.5	89.90	0.9986

The found rate constants of methylene blue adsorption were used for calculation of the adsorption values of and for construction of kinetic curves. The coefficient of correlation R^2 for the pseudo-first–order kinetic model is not high. Moreover, the determined values of am calculated from the equation (1) differ from the experimental values (Table 1). This indicates that adsorption of MB onto CCD does not follow the pseudo-first-order reaction model and had low correlation coefficient ($R^2 = 0.4630$).

Table 1. Comparison of the experimental adsorption kinetics of MB on the CCD sample with those calculated from the pseudo-second order kinetic and Elovich model

t	a_m^{exp}	a_m^{calc} (Eq. 2)	a_m^{calc} (Eq. 3)	Relativ error, $\triangle$, % (Eq. 2)
5	75.5	74.1	79.9	1.8
10	82.5	80.0	81.9	3.0
15	84.5	82.1	83.2	2.8
30	88.0	84.5	85.3	4.0
60	89.2	85.7	87.4	3.9
90	89.7	86.1	88.6	4.0
120	89.8	86.3	89.5	3.9

It can be seen from the Table 1 that the R2 for the pseudo-second-order kinetic model and model Elovich have the maximum values ($R^2 = 0.9986$ and 0.9924 respectively) and a small relative error ($\triangle, \%$) Table2; this indicates that these models most accurately describe the experimental kinetics data on the methylene blue adsorption on clayey carbonate diatomite. This gives evidence of the chemical character of the interaction of molecules MB with active centers on the sorbent surface CCD.

The analytical expressions were obtained for the pseudo-second-order equation and model Elovich, which have the following forms:

$$\begin{aligned} a_t &= \frac{t}{0.01 + 0.0115t} \\ a_t &= 3.03\,(0.01 + 0.0115 \ln t) \end{aligned} \tag{4}$$

The equations give possibility to calculate theoretically of the values of methylene blue adsorption MB on CCD for any time of the sorption process. The obtained results also show that the clayey carbonate diatomite can be used as a promising adsorbent for the removal of methylene blue from water.

Acknowledgments. *This research was supported by Project of Institute of Applied Physics ANCD 20.80009.5007.06 and by Project of Institute of Geology and Seismology 20.80009.7000.13 for the period 2020-2023*

Bibliography

[1] Y.S.Ho, *Citation review of Lagergren kinetic rate equation on adsorption reactions*, Scientometries, **59**(2004), 171–177

[2] G. McKay, Y.S. Ho, *Pseudo-second-order model for sorption processes*, Process Biochem. **34**(1999), 451–465.

[3] S.H.Chen, W.R.Clayton, *Application of Elovich equation to the kineticsof phosphate release and sorption on soils*, Soil Sci. Soc. Am. J., **44**(1980), 265-268.

4. Real, Complex, Functional and Numerical Analysis

Variable exponent systems with Leray-Lions type operators

Maria-Magdalena Boureanu

University of Craiov, Romania
e-mail: mmboureanu@yahoo.com

We discuss the weak solvability of a generalized class of anisotropic systems with variable exponents. More precisely, we are interested in obtaining the existence and multiplicity of solutions for systems with Leray-Lions type operators. We are also concerned with providing examples of systems which are particular cases of the more general system that we study.

Holder and Minkowski type inequalities in Riemann-Lebesgue integrability

Alina Iosif[1], Alina Gavriluţ[2] and Anca Croitoru[2]

[1] *Department of Computer Science, Information Technology, Mathematics and Physics, University of Ploiesti, Romania*
[2]*Faculty of Mathematics, "Alexandru Ioan Cuza" University of Iaşi, Romania*
e-mail: emilia.iosif@upg-ploiesti.ro, gavrilut@uaic.ro ,croitoru@uaic.ro

Integral inequalities have important applications in both theoretical and practical domains of mathematics. The aim of our talk is to present some integral inequalities regarding Riemann-Legesgue integrable functions in non-additive case.

Multivalued conformable differential equations

Tzanko Donchev[1],and Alina I. Lazu[2]

[1]*Department of Mathematics, UACG, 1 Hr. Smirnenski bvd, 1046 Sofia, Bulgaria,*
[2]*Department of Mathematics, "Gh. Asachi" Technical University, Iaşi 700506, Romania*
e-mail: [1]tzankodd@gmail.com, [2]vieru_alina@yahoo.com

In this paper we study first the following local Cauchy problem with conformable derivative:

$$\begin{cases} x^{(\alpha)}(t) \in F(t, x(t)),\ t \in I = [a, b], \\ x(a) = x_a, \end{cases} \tag{1}$$

where $F : I \times \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ is a given multifunction with nonempty values, $x_a \in \mathbb{R}^n$, $\alpha \in (0, 1]$, and $x^{(\alpha)}(t) = \dfrac{d^\alpha x(t)}{dt^\alpha}$ is the conformable (fractional) derivative of $x(\cdot)$ of order α.

The conformable fractional derivative was introduced in [2]. A good survey of the literature of conformable derivative and conformable differential equations is given, for example, in [1].

We show that the solution set of (1) is nonempty and compact R_δ in case of almost upper semicontinuous right-hand side. Then, an integral characterization of the solution is presented. Further, a relaxation theorem when the right-hand side satisfies the one-sided Lipschitz condition is proved. Also, we consider the nonlocal problem and provide a criteria for the existence of solutions. Finally, we provide criteria for weak and strong invariance of the solution set of (1) with respect to a closed set $\mathfrak{K}$.

Acknowledgements. The work was supported by the Bulgarian National Science Fund under Project KP-06-N32/7.

Bibliography

[1] Boukenkoul A., Ziane M, *Conformable functional evolution equations with nonlocal conditions in Banach spaces*, Survey in Math. Analysis (to appear).

[2] Khalil R., Horani M. Al, Yousef A., Sababheh M., *A new definition of fractional derivative*, J. Comput. Appl. Math. **264**, 65–70 (2014).

On some Operators on Banach Lattices Involving Demicompactness

Omer Gok

Yildiz Technical University, Istanbul, Turkey
e-mail: gok@yildiz.edu.tr

Order weakly demicompact operators, L-weakly demicompact operators, M-weakly demicompact operators, b-weakly demicompact operators, demi Dunford-Pettis operators, demi KB-operators were investigated by some authors.

In this presentation, we introduce a new class of operators on Banach lattices involving unbounded norm demicompactness and we investigate some of its properties and its relationships with some well-known classes of operators.

Bibliography

[1] H. Benkhaled, A. Jeribi,The class of demi KB-operators on Banach lattices. *Turkish J. Math.*, **47** (2023), 387-396.

[2] P. Meyer-Nieberg. *Banach lattices.* Springer-Verlag, New York, 1991.

On a class of positivity preserving linear maps in a pre-Riesz space

Cecil P. Grünfeld

"Gheorghe Mihoc-Caius Iacob" Institute of Mathematical Statistics and Applied Mathematics of Roamnian Academy, Bucharest Romania
e-mail: grunfeld_51@yahoo.com

We investigate a class of positivity preserving linear operators in a pre-Riesz space. The operators appear to be of interest in the applications of a recently introduced evolution model in a real abstract state space, in the sense of Davies, as a generalization of some nonlinear kinetic equations (e.g., Boltzmann- like equations, Smoluchowski equation). Our analysis distinguishes between the case of a pre-Riesz spaces which is actually a Banach lattice, when a complete characterization of the aforementioned positive linear maps can be obtained, and the less understood situation of a pre-Risez space which is not a (Banach) lattice, when a different picture may arise, as is revealed by an example of two-dimensional pre-Riesz space. Applications to some integro - partial differential kinetic models are also included.

Smoothing Noisy Terrain Data

Stelian Ion, Dorin Marinescu, Stefan-Gicu Cruceanu

"Gheorghe Mihoc-Caius Iacob" Institute of Mathematical Statistics and Applied Mathematics of Roamnian Academy
e-mail: stelian.ion@ismma.ro, marinescu.dorin@ismma.ro, stefan.cruceanu@ismma.ro

The main information about natural phenomena is obtained through direct measurements in environmental sciences. These data are most often irregularly distributed and affected by device or human errors. Therefore, recovering the variable field with some regular properties becomes a problem of interest. We present here a method that overcomes several difficulties when trying to solve this problem. It also offer the possibility "to cure" the presence of outliers and moderate noisy data.

On the density of Lipschitz functions in Sobolev-type spaces on metric measure spaces

Marcelina Mocanu

"Vasile Alecsandri" University of Bacău, Romania
e-mail: mmocanu@ub.ro

Lipschitz functions play the role of smooth functions in analysis on metric measure spaces, being used to approximate functions from various Sobolev spaces, of Hajlasz and Newtonian type. This approximation allows the extension of some properties, e. g. the validity of a Poincaré inequality, from Lipschitz functions to Sobolev-type functions. We summarize several results on the density of Lipschitz functions in Sobolev-type spaces on metric measure spaces, progressing from the classical case of Sobolev spaces of first order, on Euclidean domains, to the case of Sobolev-type spaces based on rearrangement invariant Banach function spaces, on metric measure spaces.

Bibliography

[1] S. Eriksson, *Bique- Density of Lipschitz functions in energy*, Calculus of Variations and Partial Differential Equations, 62 (2023), Article no. 60

[2] L. Malý, *Regularization of Newtonian functions on metric spaces via weak boundedness of maximal operators*, Journal d'Analyse Mathématique, 134 (2018), 1-54.

[3] M. Mocanu, *Density of Lipschitz functions in Orlicz-Sobolev spaces with zero boundary values on metric measure spaces*, Buletinul Institutului Politehnic din Iaşi. Secţia: Matematică, Mecanică teoretică, Fizică, tom LVII (LXI), Fasc. 1 (2011), 169-184.

[4] M. Mocanu, *Approximation by Lipschitz functions in abstract Sobolev spaces on metric spaces*, Mathematical Reports, 15 (65), no. 4, 459-475, 2013

An adaptive time step method for a non-local version of the Allen-Cahn equation

A.-M. Moșneagu[1,2,3,4] and I. Stoleriu[1,5]

[1] ”"Alexandru Ioan Cuza" University of Iaşi, Faculty of Mathematics,Iaşi, Romania;
[2] "Alexandru Ioan Cuza" University of Iaşi, Research Center with Integrated Techniques for Atmospheric Aerosol Investigation in Romania, RECENT AIR,
[3] Laboratory of astronomy and astrophysics,Iasi, Romania
e-mail: [4]anamaria.mosneagu@uaic.ro, [5]iulian.stoleriu@uaic.ro

In this paper, we propose an efficient adaptive time step method for a non-local version of the Allen-Cahn equation. The non-local model was introduced as an alternative model for phase transitions in solids. This model becomes very useful when the transition process operates at very small length scales, case in which its local correspondent could not be applied. Some computational experiments are made to illustrate the effectiveness of the proposed method, with the goal of saving the calculation costs.

New results in the theories of third-order differential subordinations and superordinations

Lavinia Florina Preluca[1,2] and Georgia Irina Oros[1,3]

[1] University of Oradea,
[2] Doctoral School of Engineering Sciences,
[3] Department of Mathematics and Computer Science, Faculty of Informatics and Science Oradea, Romania
e-mail: [2]preluca.laviniaflorina@student.uoradea.ro, [3]georgia_oros_ro@yahoo.co.uk

The idea of differential subordination was first presented by S.S. Miller and P.T. Mocanu in two works that were published in 1978 and 1981 in an effort to extend the concept of inequality from the real numbers to the complex plane. S.S. Miller and P.T. Mocanu developed the theory of differential subordination in the context of second-order differential subordinations [4]. In 2011, J.A. Antonino and S.S. Miller [1] developed this idea to include third-order differential subordinations, providing opportunities for further research into the differential subordination theory. This area of research focuses on identifying previously established findings from the theory of second-order differential subordinations that, with the proper extension, are also applicable to the theory of third-order differential subordinations. The initial such extensions could be realized using one of the core concepts of the theory of differential subordinations, the class of admissible functions, as a starting point. The results presented here concerning third-order differential subordinations were obtained considering another fundamental problem in differential subordination theory, which is identifying dominants for the differential subordinations studied and furthermore, finding the best dominant when this is possible [7, 8]. The fractional integral of Gaussian hypergeometric function [6] is used for providing applications of the theoretical results obtained. The next step accomplished in 2014 [9] was to expand the dual theory of differential superordination [3, 2] in order to include third-order differential superordination. New and intriguing findings followed soon considering the idea involving the class of admissible functions [10] and continues to present interest at this time [11]. The extension of the results established for second-order differential superordiantions is now presented following the idea of finding subordinants of the third-order differential superordinations and providing the best subordinant when admitted by the third-order differential superordinations involved in the study.

Keywords. Analytic function; convex function; third-order differential subordination; best dominant; third-order differential superordination; best subordinant; subordination chain; fractional integral; Gaussian hypergeometric function.

2020 Mathematics Subject Classification: 30C80, 33C15, 30C45.

Bibliography

[1] J.A. Antonino, S.S. Miller, *Third-order differential inequalities and subordinations in the complex plane,* Complex Var. Elliptic Equ., **56(5)** (2011), 439-454. https://doi.org/10.1080/17476931003728404

[2] T. Bulboacă, *Differential Subordinations and Superordinations: Recent Results*, House of Scientific Book Publishing, Cluj-Napoca, Romania, 2005.

[3] S.S. Miller, P.T. Mocanu, *Subordinates of differential superordinations*, Complex Var. Theory Appl., **48** (2003), 815-826.

[4] S.S. Miller, P.T. Mocanu, *Differential Subordinations. Theory and Applications*, Marcel Dekker Inc., New York, Basel, 2000.

[5] A.K. Mishra, A. Prajapati, P. Gochhayat, *Third order differential subordination and superordination results for analytic functions involving the Hohlov operator*, Tbilisi Mathematical Journal, **13(3)** (2020), 95-109.

[6] G.I. Oros, S. Dzitac, *Applications of Subordination Chains and Fractional Integral in Fuzzy Differential Subordinations*, Mathematics, **10 (10)** (2022), 1690. https://doi.org/10.3390/math10101690

[7] G.I. Oros, G. Oros, L.F. Preluca, *Third-Order Differential Subordinations Using Fractional Integral of Gaussian Hypergeometric Function*, Axioms, **12** (2023), 133. https://doi.org/10.3390/axioms12020133

[8] G.I. Oros, G. Oros, L.F. Preluca, *New applications of Gaussian hypergeometric function for developments on third-order differential subordinations*, Symmetry, **15 (7)** (2023), 1306. https://doi.org/10.3390/sym15071306

[9] H. Tang, H.M. Srivastava, E. Deniz, S.-H. Li, *Third-Order Differential Superordination Involving the Generalized Bessel Functions*, Bull. Malays. Math. Sci. Soc., **38** (2014), 1669–1688.

[10] H. Tang, H.M. Srivastava, S.-H. Li, L. Ma, *Third-Order Differential Subordination and Superordination Results for Mero-morphically Multivalent Functions Associated with the Liu-Srivastava Operator*, Abstr. Appl. Anal., **2014** (2014), 1–11.

[11] S.D. Theyab, W.G. Atshan, A. Alb Lupaș, H.K. Abdullah, *New Results on Higher-Order Differential Subordination and Su-perordination for Univalent Analytic Functions Using a New Operator*, Symmetry, **14** (2022), 1576. https://doi.org/10.3390/sym14081576

Perturbations of Rhaly Operators

George Popescu

University of Craiova, Romania
e-mail: grgpop@gmail.com

We study Rhaly operators on separable Hilbert spaces. Such operators are defined by terraced matrices, generated by a sequence of complex numbers.

Some properties of the defining sequence, imply boundedness or compactness of Rhaly operators. We may call such properties - regularity properties.

We prove that perturbations of the defining sequence, namely replacing just one subsequence, may preserve boundedness or compactness, only if the subsequence is rare.

Splines for Set Functions

Vasile Postolică

Romanian Academy of Scientists,
"Vasile Alecsandri" University of Bacău,
Faculty of Sciences,
Department of Mathematics and Informatics,
Romania
e-mail: vasilepostolica55@gmail.com

This research paper deals with the splines induced by an original method which generates, in particular, the best simultaneous (vectorial) approximation and optimal interpolation elements in any H-locally convex space, that is, in every Hausdorff locally convex space with each semi-norm satisfying the parallelogram law, applied to a class of set functions. In such a way as this, we indicate again our first procedure which extends, in a natural manner, the Best Approximation Problem solved by the usual spline functions in Hilbert spaces to the H-locally convex spaces.

Mathematics Subject Classification (2000): 41A65.

Keywords: spline function, H-locally convex space, best approximation, optimal interpolation, set function.

Bibliography

[1] Isac, G., Postolică, V., *The Best Approximation and Optimization in Locally Convex Spaces*, Verlag Peter Lang GmbH, Frankfurt am Main, Germany, 1993.

[2] Postolică, V., *A Method which Generates Splines* in H - Locally Convex Spaces and Connections with Vectorial Optimization, Positivity, 2 (4), 1998, p. 369 - 377.

[3] Postolică V.,*Spline Approximation of Countable Additive Set Functions of Bounded Variation* Communication to The Third International Symposium on Orthogonal Polynomials and Their Applicatons Erice, Italy, June 1 - 8, 1990. Published by "Babeş – Bolyai" University, Cluj – Napoca, Romania, Faculty of Mathematics and Informatics, Seminar on Mathematical Analysis, Preprint 7/1990, p. 63 – 76.

[4] Postolică V., *On the Best Approximation of Set Functions*, Proceeding of The Fourth International Conference on Functional Analysis and Approximation Theory, Acquafreda di Maratea, Potenza - Italy, September 22 - 28, 2000. Published in Supplemento ai Rendiconti del Circolo Matematico di Palermo, serie II, no 68, 2002, p. 761 - 775.

[5] Postolică, V., *Isac's Cones in General Vector Spaces*, Published as Chapter 121 of Encyclopedia of Business Analytics and Optimization 5 Vols, IGI Global, 2014, p. 1323 – 1342.

[6] Precupanu, A. M., Precupanu, T., *Some Kinds of Best Approximaion Problems in a Locally Convex Space*, An. Şt. Univ. "Al. I. Cuza" Iasi, Romania, XLVIII, 2002, p. 409 - 421.

A study on the viability of Stieltjes differential inclusions

Bianca Satco

"Stefan cel Mare" University of Suceava, Romania
e-mail: bisatco@usm.ro

The aim of the talk is to present a viability result ([3]) for differential inclusions involving the Stieltjes derivative ([1]) with respect to a left-continuous non-decreasing function $g : [0,1] \to \mathbb{R}$

$$\begin{cases} x'_g(t) \in F(t, x(t)),\ \mu_g - a.e.\ t \in [0,1) \\ x(t) \in K(t), \forall\, t \in [0,1] \\ x(0) = x_0 \in K(0) \end{cases}$$

with time dependent state constraints. The main tangential condition involves the notion of contingent g-derivative ([2]).
We show that classical results are thus covered and new results are inferred for dynamic problems on time scales and for impulsive differential problems.

Bibliography

[1] R.L. Pouso, A. Rodriguez, *A new unification of continuous, discrete, and impulsive calculus through Stieltjes derivatives*, Real Anal. Exch., 40(2015) 319–353.

[2] R. L. Pouso, I. Marquez Albes, J. Rodriguez-Lopez, *Solvability of non-semicontinuous systems of Stieltjes differential inclusions and equations*, Adv. Differ. Equ., 227 (2020)(2020) 1-14.

[3] B. Satco, G. Smyrlis, *Viability and Filippov-type lemma for Stieltjes differential inclusions*, Set-valued and variational analysis, to appear.

On estimation of weakly singular integral operators and theirs applications in approximate solving of weakly singular integral equations

Vladislav Seichiuc, Eleonora Seichiuc, Gheorghe Carmocanu

Technical University of Moldova, Moldova State University,
Chişinău, Republic of Moldova
e-mail: vladislav.seiciuc@calc.utm.md, eleonora.seiciuc@usm.md, gcarmocanu@yahoo.com

Implementation of approximate methods for solving weakly singular integral equations (WSIE) of second kind leads to the necessity of concrete estimation in Banach spaces of Fredholm and

Volterra weakly singular integral operators (WSIO) and their modifications that appear in proposed algorithms.

In this work, at first, concrete values of the constants that estimate the Fredholm and Volterra WSIO in the spaces of continuous function and Hölder spaces are obtained.

Then, based on the obtained results, we give theoretical foundation in the spaces of continuous function and the Hölder spaces for collocation, quadratures, splin-collocation and splin-quadratures methods in solving WSIE of second kind. The theoretical foundation assumes:

1) Replacing the WSIE core with a truncated core so that the transformed WSEI remains compatible;

2) The estimation relations of the WSIE solutions with truncated kernel, compared to the solutions of the initial equation, are known;

3) Moving to discrete domains, so that the proposed approximate methods applied to truncated kernel WSIEs are compatible and convergent;

4) For each method, the estimation relations of the calculated approximate solutions are obtained.

Duality Results for a Class of Control Problems with Partial Differential Inclusions

Sevilay Demir Sağlam

Istanbul University, Department of Mathematics, Istanbul, Turkey
e-mail: `sevilay.demir@istanbul.edu.tr`

One can apply the differential calculus of set-valued mappings to find global set-valued solutions to partial differential equations and inclusions. Convexity and duality first appear in the classical calculus of variations while examining the connection between Lagrangian and Hamiltonian functions and how this relates to the existence of solutions and necessary conditions. We point out that a significant part of the research of Ekeland and Temam [4] for simple variational problems is connected with such problems and the results are also valid for ordinary differential inclusions. Duality theory is one of the main approaches to convex optimality problems due to the importance of its applications, and it is understood differently for different concrete circumstances. The study of economic models, in which dual variables can be thought of as prices and can be helpful in various computing approaches, is one theoretical use of this duality. In this study, We demonstrate that the duality theorems allow one to conclude that a sufficient condition for an extremum is an extremal relation for the primal and dual problems. We have two objectives; to demonstrate optimality conditions for polyhedral problems and to define sufficient conditions of optimality for partial differential inclusions for convex problems. Moreover, to construct the dual problem to the optimal control problem presented by differential inclusions and prove duality relations.

Mathematics Subject Classification: 49K20, 34A60, 49N15, 90C46.

Keywords: differential inclusion; duality; optimality conditions.

Bibliography

[1] Cernea, A.,*Some second-order necessary conditions for nonconvex hyperbolic differential inclusion problem*, J. Math. Anal. Appl. 253(2), 616–639 (2001).

[2] Demir Sağlam, S., Mahmudov, E.N., *The Lagrange Problem for Differential Inclusions with Boundary Value Conditions and Duality*, Pacific J. Optim. 17(2), 209-225 (2021).

[3] Demir Sağlam S, *Polyhedral optimization of discrete and partial differential inclusions of parabolic type*, Optimization, 2023, DOI: 10.1080/02331934.2023.2199032

[4] Ekeland, I., Temam, R., *Convex Analysis and Variational Problems*, North-Holland, Amsterdam, The Netherlands (1976).

[5] Mahmudov, E.N., *Approximation and Optimization of Discrete and Differential Inclusions*, Elsevier, Boston, USA (2011).

[6] Mordukhovich, B.S.: Raymond JP., *Neumann boundary control of hyperbolic equations with pointwise state constraints*, SIAM J. Control Optim. 43(4), 1354-1372 (2004).

[7] Rockafellar, R.T., Wolenski, P.R., *Convexity in Hamilton-Jacobi theory I: dynamics and duality* SIAM J. Control Optim. 39, 1323-1350 (2000).

Generalized convexity and applications

Cristina Stamate

"Octav Mayer" Institute of Mathematics, Romanian Academy,
Iaşi Branch, Bd. Carol I, No. 11, Iaşi 700506, Romania
e-mail: `cstamate@ymail.com`

Following the Hahn-Banach theorem for convex sets we introduce and study a type of generalized convexity for sets. Several results concerning the separation theorems for nonconvex sets are presented.

Positive solutions for a fractional differential equation with sequential derivatives and nonlocal boundary conditions

Alexandru Tudorache[1] , Rodica Luca Tudorache[2]

[1]*Department of Computer Science and Engineering,*
[2]*Department of Mathematics,*
"Gheorghe Asachi" Technical University, Iasi 700050, Romania
e-mail: [1]`alexandru-gabriel.tudorache@academic.tuiasi.ro`, [2]`rluca@math.tuiasi.ro`

We study the existence of positive solutions for a Riemann-Liouville fractional differential equation with sequential derivatives, a positive parameter and a sign-changing singular nonlinearity, supplemented with nonlocal boundary conditions which contain various fractional derivatives and Riemann-Stieltjes integrals. In the proof of the main results, we apply the Guo-Krasnosel'skii fixed point theorem, (see [1]).

Bibliography

[1] A. Tudorache, R. Luca, *Positive solutions for a fractional differential equation with sequential derivatives and nonlocal boundary conditions*, Symmetry, **14** (1779), (2022), 1-12.

5. Probability Theory, Mathematical Statistics, Operations Research

Some statistical tests based on divergence measures

Vlad Stefan Barbu

Laboratory of Mathematics Raphael Salem, University of Rouen – Normandy, France
"Vladimir Trebici" Centre for Demographic Research
"Costin C. Kiritescu" National Institute of Economic Research of Romanian Academy, Romania
e-mail: barbu@univ-rouen.fr

Divergence measures are of great importance in statistical inference; equally important are their limiting versions, known as divergence rates. In the first part of our presentation, we focus on generalized divergence measures for Markov chains. We consider generalizations of Alpha divergence measure (Amari and Nagaoka, 2000) and Beta divergence measures (Basu et. al, 1998) and investigate their limiting behaviour. We also study the corresponding weighted generalized divergence measures and the associated rates (Belis and Guiasu, 1968; Guiasu, 1971; Kapur, 1994).

In the second part of our presentation, we focus on hypothesis testing based on weighted divergences. More precisely, we present a goodness of fit test and a homogeneity test and we study their performance. This type of tests based on weighted divergences allow us to focus on specific subsets of the support without, at the same time, losing the information of the others. With this method we achieve a significantly more sensitive test than the classical ones but with comparable error rates.

Keywords: Divergence measures, weighted divergence measures, entropy, Markov processes, hypotheses testing

Bibliography

[1] V. S. Barbu, A. Karagrigoriou, V. Preda, *Entropy and divergence rates for Markov chains: III. The Cressie and Read case and applications*, Proceedings of the Romanian Academy-series A: Mathematics, Physics, Technical Sciences, Information Science, **19**(3), 413-421, 2018.

[2] V. S. Barbu, A. Karagrigoriou, V. Preda, *Entropy and divergence rates for Markov chains: II. The weighted case* , Proceedings of the Romanian Academy-series A: Mathematics, Physics, Technical Sciences, Information Science, **19**(1), 3-10, 2018.

[3] S. Barbu, A. Karagrigoriou, V. Preda, *Entropy and divergence rates for Markov chains: I. The Alpha-Gamma and Beta-Gamma case* , Proceedings of the Romanian Academy-series A: Mathematics, Physics, Technical Sciences, Information Science, **18**(4), 293-301, 2017.

[4] T. Gkelsinis, A. Karagrigoriou, V. S. Barbu, *Statistical inference based on weighted divergence measures with simulations and applications* , Statistical Papers, 282, 1-25, 2022. DOI: https://doi.org/10.1007/s00362-022-01286-z

A production planning model of reconfigurable manufacturing lines

Buzatu Radu

Moldova State University, Chişinău, Republic of Moldova
e-mail: radu.buzatu@usm.md, radubuzatu@gmail.com

Today manufacturing companies are facing a very dynamic environment, and they have to use reconfigurable manufacturing lines (RML) to stay competitive. Current production planning models oriented on dedicated manufacturing lines do not allow to produce of more than one product per line without costly changes in manufacturing and are limited in flexibility. A novel production planning model of RML is presented in this paper. This model provides a high degree of pliancy for a wide range of product types to be manufactured.

The main characteristics and planning requirements for RML are defined, taking into account manufactured products, manufacturing periods, line configurations, demand scenarios, and different model parameters like production costs, reconfiguration costs, idle costs, and others. The main goal is to determine the minimum-cost configuration for a target production program.

An optimal configuration of RML was determined, using mixed integer linear programming (MILP) to realize capacity scalability and functionality changes within planning processes. Finally, the model was validated in a scenario based on an industrial use case.

Solving informational extended bimatrix games

Hâncu Boris

Moldova State University, Chişinău, Republic of Moldova
e-mail: boris.hincu@usm.md

Bimatrix game of imperfect information on the set of informational extended strategies generates the normal form incomplete information game

$$\widetilde{\mathbf{\Gamma}} = \left\langle \{1,2\}, I, J, \left\{ AB(\alpha,\beta) = \left\| \left(a_{ij}^{\alpha\beta}, b_{ij}^{\alpha\beta} \right) \right\|_{i\in I}^{j\in J} \right\}_{a=\overline{1,t}}^{\beta=\overline{1.k}} \right\rangle .$$

We can construct the bimatrix Bayesian game for the game $\widetilde{\mathbf{\Gamma}}$ that consists of the following: **1)** set of players $\{1,2\}$; **2)** a set of possible actions for each player: for player 1 is $I = \{1,2,..,n\}$, the line index, and for player 2 is $J = \{1,2,..,m\}$, the column index; **3)** a set of possible types of the player 1 are $\Delta_1 = \{\alpha = 1,...,t\}$ and of the player 2 are $\Delta_2 = \{\beta = 1,...,k\}$,were only player 1 (player 2) knows his type α (type β) when play begins; **4)** a probability function that specifies, for each possible type of each player, a probability distribution over the other player's possible types, describing what each type of each player would believe about the other players' types $p : \Delta_1 \to \Omega(\Delta_2)$, $q : \Delta_2 \to \Omega(\Delta_1)$,where $\Omega(\Delta_2)$ (respectively $\Omega(\Delta_1)$) denotes the set of all probability distributions on a set Δ_1 (respectively Δ_2); **5)** combining actions and types for each player we construct the strategies $\tilde{\imath} = i_1 i_2 ... i_\beta ... i_k \in \widetilde{I}(\alpha)$ of the player 1 ($\widetilde{j} = j_1 j_2 ... j_\alpha ... j_t \in \widetilde{J}(\beta)$ of the player 2) and it has the following meaning: the player 1 will chose the line $i_1 \in I$ from the utility matrix $A(\alpha,1)$ if $\beta = 1$, the line $i_2 \in I$ from the utility matrix $A(\alpha,2)$ if $\beta = 2$ and so on, line $i_k \in I$ from the utility matrix $A(\alpha,k)$ if $\beta = k$ (the player 2 will chose column $j_1 \in J$ from utility matrix $B(1,\beta)$ if $\alpha = 1$, column $j_2 \in J$ from utility matrix $B(2,\beta)$ if $\alpha = 2$ and so on, column $j_t \in J$ from utility matrix $B(t,\beta)$ if $\alpha = t$; **6)** a payoff function specifies each player's expected payoff matrices for every possible combination of all player's actions and types, if the player 1 of type α chooses the pure strategy $\widetilde{i} \in \widetilde{I}(\alpha)$, and the player 2 plays some strategy $\widetilde{j} \in \widetilde{J}(\beta)$ for all $\beta \in \Delta_2$, then expected payoffs of player 1 is the following matrix $\mathbf{A}(a) = \left\|\mathbf{a}_{\widetilde{\mathbf{i}}\widetilde{\mathbf{j}}}\right\|_{\widetilde{\mathbf{i}}\in\widetilde{\mathbf{I}}(a)}^{\widetilde{\mathbf{j}}\in\widetilde{\mathbf{J}}(\beta)}$ where $\mathbf{a}_{\widetilde{\mathbf{i}}\widetilde{\mathbf{j}}} = \sum_{\beta\in\Delta_2} p(\beta/a)a_{ij}^{\alpha\beta}$(similarly, if player 2 of type β chooses the pure strategy $\widetilde{j} \in \widetilde{J}(\beta)$ and the player 1 plays some strategy $\widetilde{i} \in \widetilde{I}(\alpha)$ for all $\alpha \in \Delta_1$, then expected payoffs of player 2 of type β is $\mathbf{B}(\beta) = \left\|\mathbf{b}_{\widetilde{\mathbf{i}}\widetilde{\mathbf{j}}}\right\|_{\widetilde{\mathbf{i}}\in\widetilde{\mathbf{I}}(a)}^{\widetilde{\mathbf{j}}\in\widetilde{\mathbf{J}}(\beta)}$ where $\mathbf{b}_{\widetilde{\mathbf{i}}\widetilde{\mathbf{j}}} = \sum_{a\in\Delta_1} q(\alpha/\beta)b_{ij}^{\alpha\beta}$.

So for the game $\widetilde{\mathbf{\Gamma}}$ the game $\Gamma_{Bayes} = \left\langle \{1,2\}, \widetilde{\mathbf{I}}, \widetilde{\mathbf{J}}, \mathcal{A}, \mathcal{B} \right\rangle$, where $\widetilde{\mathbf{I}} = \bigcup_{\alpha\in\Delta_1} \widetilde{I}(\alpha)$, $\widetilde{\mathbf{J}} = \bigcup_{\beta\in\Delta 2} \widetilde{J}(\beta)$ and the utility matrices are $\mathcal{A} = \|\mathbf{A}(a)\|_{\alpha\in\Delta_1}$ and $\mathcal{B} = \|\mathbf{B}(\beta)\|_{\beta\in\Delta_2}$, is called the associated Bayesian game in the non informational extended strategies.

Using given above constructions and the Harsanyi theorem we can prove following theorem.

Theorem 1 *The strategy profile* $(\mathbf{i}^*, \mathbf{j}^*)$ *is a Bayes-Nash equilibrium in the game* Γ_{Bayes} *if and only if, for all* $\alpha \in \Delta_1, \beta \in \Delta_2$, *the strategy profile* $(\mathbf{i}^*, \mathbf{j}^*)$ *is a Nash equilibrium for the subgame* $sub\Gamma_{Bayes} = \left\langle \{1,2\}, \widetilde{\mathbf{I}}(\alpha), \widetilde{\mathbf{J}}(\beta), \mathbf{A}(\alpha), \mathbf{B}(\beta) \right\rangle$.

On k-König-Egerváry graphs

Eugen Mandrescu[1]
joint work with Vadim E. Levit[2]

[1] *Holon Institute of Technology, Israel,*
[2] *Ariel University, Israel*
e-mail: [1]eugen_m@hit.ac.il

Let $\alpha(G)$ denote the cardinality of a maximum independent set, while $\mu(G)$ be the size of a maximum matching in graph $G = (V, E)$. If $\alpha(G) + \mu(G) = |V|$, then G is a *König-Egerváry graph*, while if $\alpha(G) + \mu(G) = |V| - k$, then G is a *k-König-Egerváry graph.* 1-König-Egerváry graphs are also known as almost König-Egerváry graphs. If G is not König-Egerváry, but there exists a vertex $v \in V$ (an edge $e \in E$) such that $G - v$ $(G - e)$ is König-Egerváry, then G is called a vertex almost König-Egerváry (an edge almost König-Egerváry graph, respectively).

The presentation mostly concerns several interrelations between structural properties of almost (vertex / edge) König-Egerváry graphs.

On a new order relation between random variables related to investment problems

Ana Maria Raducan[1], Raluca Vernic[2], Gheorghita Zbaganu[1],

[1] *"Gheorghe Mihoc-Caius Iacob" Institute of Mathematical Statistics and Applied Mathematics of Romanian Academy, Bucharest, Romania*
[2] *"Ovidius" University of Constanta, Romania*
e-mail: anaraducan@yahoo.ca

Related to a stochastic investment problem which aims to determine when is it better to first invest a larger amount of money and afterwards a smaller one, in this paper we introduce a new preference relation between random variables. We investigate the link between this new relation and some well-known stochastic order relations and present some characterization properties illustrated with numerical examples.

Indices of inequality, compatibility conditions

Gheorghiță Zbăganu

"Gheorghe Mihoc-Caius Iacob" Institute of Mathematical Statistics and Applied Mathematics of Romanian Academy, Bucharest, Romania
e-mail: gheorghitazbaganu@yahoo.com

There are many inequality metrics concerning the incomes or wealth of a population. According to some of them a population P_1 exhibits more inequality that the population P_2, according to others the converse occurs.

We want to find compatibility conditions between three of them: the Gini index, the R/P 20:20 and the R/P 10:10.

All of them are related to the Lorenz curves of the populations.

If the incomes of a population have the probability distribution function F, where $F(x)$ means the probability that an member of it gains less than x monetary units at a fixed time interval (usually years or months), then its Lorenz curve is given by

$$L_F(p) = \frac{\int_0^p F^{-1}(x)\,dx}{\int_0^1 F^{-1}(x)\,dx}.$$

If $L_{F_1} \leq L_{F_2}$ the meaning is that for any $p \in (0,1)$ the proportion of the total incomes gained by the most poor p members from F_1 is smaller than the same proportion in population F_2, meaning that F_1 exhibits more inequality than F_2.

In general, the Lorenz curves of different populations intersect each other, hence one cannot say that the one is more egalitarian than the other.

That is why in order to make tops of the inequalities, people prefer to use numeric indices. There are at least four of them used in sociology, economics, demography:

1. $\text{Gini}(F) = 1 - 2\int_0^1 L_F(p)\,dp$ is the double of the area between L and the identical function;
2. $\text{R/P 20:20 }(F) = \dfrac{1 - L_F(0.8)}{L_F(0.2)}$ is the ratio between the gains of the most rich 20% and the most poor 20%. R/P is abbreviation for Rich/Poor;
3. $\text{R/P 10:10}(F) = \dfrac{1 - L_F(0.9)}{L_F(0.1)}$ is the ratio between the gains of the most rich 10% and the most poor 10%. R/P is abbreviation for Rich/Poor;
4. $\text{Palma}(F) = \dfrac{1 - L_F(0.9)}{L_F(0.4)}$ is the ratio between the gains of the most rich 10% and the most poor 40%.

Gini index is the most popular because it lies between 0 and 1. The Palma index is used more and more often in the last years because it is more stable from a statistical point of view.

The goal is to study the relations between these four indicators of inequality: Gini, R/P 20:20,RP 10:10,Palma.

The maximal goal was to describe in the set
$\{(\text{Gini}(F), \text{R/P } 20{:}20(F), \text{RP } 10{:}10(F), \text{Palma}(F) : F$ probability distribution on $[0,\infty)\}$
It seems to be a polyhedron in $\mathbb{R}^4$.We were not able to describe it.
The next one is to give up Palma(F) and to describe the set
$M = \{(\text{Gini}\,(F)\,, \text{R/P20:20}\,(F)\,, \text{RP10:10}\,(F) : F$ probability distribution on$[0,\infty)\}$
Thus we find bounds for Gini(F) when $k\,(F)$ =R/P20:20(F) and $K\,(F)$ =R/P20:20(F) are given and conversely, bounds for $k\,(F)$ and $K\,(F)$ when Gini(F) is given.
Many problems remain unsolved.

Bibliography

[1] Cowell, Frank, *Measuring Inequality*, 3rd edn, Oxford University Press, 2011

6. Algebra, Logic, Geometry (with applications)

Ribbon structures and Lie pairs

Cristian Anghel
joint project with Dorin Cheptea

"Simion Stoilov" Institute of Mathematics of the Romanian Academy, Bucharest, Romania
e-mail: cristian.anghel@imar.ro

Ribbon structures in derived categories of coherent sheaves on hyperkahler manifolds, appeared for the first time in the work of Roberts and Willerton, concerning Lie type structures in Rozansky-Witten theories. After that, part of this approach was extended to symplectic Lie pairs by Voglaire-Xu and by Chen-Stienon-Xu. We intend to review this topic and discuss the existence of ribbon structures in the general context of symplectic Lie pairs.

Some six-dimensional planar submanifolds of Cayley algebra of constant type

Galia Banaru

Smolensk State University, Russia
e-mail: mihail.banaru@yahoo.com

1. As it is known, an almost Hermitian manifolds is an even-dimensional manifold M^{2n} equipped with a Riemannian metric $g = \langle \cdot, \cdot \rangle$ and an almost complex structure J. Moreover, the following condition must hold

$$\langle JX,\ JY \rangle = \langle X,\ Y \rangle\,, \quad X, Y \in \aleph(M^{2n}),$$

where $\aleph(M^{2n})$ is the module of smooth vector fields on the manifold M^{2n}. An almost Hermitian manifold is Hermitian, if its almost complex structure is integrable [1].

It is also known that the Gray–Brown 3-fold vector cross products [2] in Cayley algebra gives a set of substantive examples of six-dimensional almost Hermitian manifolds [3], [4]. Such almost Hermitian structures (in particular, Kählerian, nearly Kählerian, quasi Kählerian, Hermitian and special Hermitian) on six-dimensional submanifolds of Cayley algebra were studied by a number of outstanding geometers: E. Calabi, A. Gray, V. F. Kirichenko, K. Sekigawa and L. Vranchen.

We consider the notion of type constancy for such six-dimensional almost Hermitian planar submanifolds of Cayley algebra. We select the case when six-dimensional submanifolds of Cayley algebra are locally symmetric. Remark that the notion of type constancy was introduced by A. Gray for nearly Kählerian manifolds [5] and later it was generalized by V. F. Kirichenko and I. V. Tret¡92¿yakova for arbitrary almost Hermitian manifolds [6]: an almost Hermitian manifold is called of constant type c if

$$\| \, N_J \, (X, Y) \, \| = c \, \|X\|^2 \|Y\|^2,$$

where N_J is the Nijenhuis tensor of the operator of almost complex structure.

2. The main result of the present communication is the following:

Theorem. *Six-dimensional locally symmetric submanifolds of Ricci type of Cayley algebra are almost Hermitian manifolds of zero constant type.*

This result means that six-dimensional locally symmetric submanifolds of Ricci type of Cayley algebra demonstrate a property of six-dimensional Kählerian submanifolds of Cayley algebra. However, there exist non-K̈ahlerian six-dimensional locally symmetric submanifolds of Ricci type in Cayley algebra [7], [8].

Bibliography

[1] V. F. Kirichenko, *Differential-geometric structures on manifolds*, Pechatnyi Dom, Odessa, (2013) (in Russian).

[2] R. Brown, A. Gray, *Vector cross products*, Comm. Math. Helv., 42 (1967), 222–236.

[3] A. Gray, *Six-dimensional almost complex manifolds defined by means of three-fold vector cross products*, Tôhoku Math. J., 21 (1969), 614–620.

[4] V. F. Kirichenko, *Hermitian geometry of six-dimensional symmetric submanifolds of Cayley algebra*, Mosc. Univ. Math. Bull., 49 (3) (1994), 4–9.

[5] A. Gray, *Nearly Kähler manifolds*, J. Diff. Geom., 4 (1970), 283–309.

[6] V. F. Kirichenko, I. V. Tret'yakova, *On the constant type of almost Hermitian manifolds*, Math. Notes, 68 (5) (2000), 569–575.

[7] M. B. Banaru, G. A. Banaru,*A note on six-dimensional planar Hermitian submanifolds of Cayley algebra*, Buletinul Academiei de Ştiinţe a Republicii Moldova. Matematica, 74 (1) (2014), 23–32.

[8] M. B. Banaru, *Geometry of 6-dimensional Hermitian manifolds of the octave algebra*, J. Math. Sci., New York, 207 (3) (2015), 354–388.

On quasi-Sasakian structures on η-quasi-umbilical hypersurfaces in QK-manifolds

Mihail Banaru

Smolensk State University, Russia
e-mail: mihail.banaru@yahoo.com

1. Structures induced on an oriented hypersurface in an almost Hermitian manifold are the most important examples of almost contact metric structures [1], [2]. We recall that an almost contact metric structure on an odd-dimension manifold N^{2n-1} is defined by a system of tensor fields $\{\Phi,\ \xi,\ \eta,\ g\}$ on this manifold. Here ξ is a vector, η is a covector, Φ is a tensor of the type $(1,1)$ and $g = \langle\cdot,\ \cdot\rangle$ is the Riemannian metric. Moreover, the following conditions are fulfilled:

$$\eta(\xi) = 1,\ \Phi(\xi) = 0,\ \eta\circ\Phi = 0,\ \Phi^2 = -id + \xi\otimes\eta;$$

$$\langle\Phi X, \Phi Y\rangle = \langle X, Y\rangle - \eta(X)\eta(Y),\ \ X, Y \in \aleph(N^{2n-1}).$$

where $\aleph(N^{2n-1})$ is the module of C^∞-smooth vector fields on the manifold N^{2n-1} [1]. In accordance with the definition [1], an almost contact metric structure is Sasakian, if

$$\nabla_X(\Phi)Y = \langle X, Y\rangle\,\xi - \eta(Y)X,\ \ X, Y \in \aleph(N^{2n-1}).$$

As it is well known, Sasakian structures play a fundamental role in contact geometry and theoretical physics. A natural generalization of the Sasakian structure is the quasi-Sasakian structure. An almost contact metric structure $\{\Phi,\ \xi,\ \eta,\ g\}$ is called quasi-Sasakian if its fundamental form $\Omega(X,Y) = \langle X,\ \Phi Y\rangle$ is closed and the following condition holds:

$$N_\Phi + \frac{1}{2}\,d\eta\otimes\xi = 0$$

where N_Φ is the Nijenhuis tensor of the operator Φ. The theory of quasi-Sasakian structures was created by the outstanding American geometer D.E. Blair [3]. We remark also that the start point of the study of quasi-Sasakian structures on hypersurfaces in almost Hermitian manifolds is the work by H. Yanamoto [4].

2. In the present communication, quasi-Sasakian structures on hypersurfaces in quasi-Kählerian manifolds (QK-manifolds, or $W_1 \oplus W_2$-manifolds, using Gray–Hervella notation [5]) are considered. We recall that a hypersurface N^{2n-1} of a qusi-Kählerian manifold M^{2n} is called η-quasi-umbilical if the second fundamental form of the immersion of N^{2n-1} into M^{2n} looks as follows:

$$\sigma\,(X,Y) \;=\; \lambda\,\langle X,Y\rangle + h\,\eta\,(X)\,\eta\,(Y),$$

where $h = \sigma(\xi,\, xi)$, λ — const. Evidently, if $h = 0$, then $\sigma(X,\, Y) = \lambda\langle X,\, Y\rangle$, that is why in this case N^{2n-1} is a totally umbilical hypersurface of M^{2n}. If $h = 0$ and $\lambda = 0$, then $\sigma(X,\, Y) = 0$, so the hypersurface N^{2n-1} is totally geodesic.

The following two theorems contain the main result of this communication.

Theorem A. *The first group of the Cartan structural equations of the quasi-Sasakian structure on an oriented η-quasi-umbilical hypersurface N^{2n-1} in a quasi-Kählerian manifold M^{2n} looks as follows:*

$$d\omega^\alpha = \omega^\alpha_\beta \wedge \omega^\beta - i\lambda\,\delta^\alpha_\beta\omega \wedge \omega^\beta;$$

$$d\omega_\alpha = -\omega^\beta_\alpha \wedge \omega_\beta + i\lambda\,\delta^\beta_\alpha\omega \wedge \omega_\beta;$$

$$d\omega = -2\,i\lambda\,\omega^\beta \wedge \omega_\alpha.$$

Theorem B. *The quasi-Sasakian structure on an oriented η-quasi-umbilical hypersurface N^{2n-1} in a quasi-Kählerian manifold M^{2n} is either cosymplectic or homothetic to the Sasakian structure.*

We remark that this communication is a continuation of the researches of the author, who studied diverse properties of Sasakian and quasi-Sasakian structures on oriented hypersurfaces in almost Hermitian manifolds [6], [7], [8], [9].

Bibliography

[1] V. F. Kirichenko, *Differential-geometric structures on manifolds*, Pechatnyi Dom, Odessa, (2013) (in Russian).

[2] M. B. Banaru, V. F. Kirichenko, *Almost contact metric structures on the hypersurface of almost Hermitian manifolds*, J. Math. Sci., New York, 207 (4) (2015), 513–537.

[3] D. E. Blair, *The theory of quasi-Sasakian structures*, J. Diff. Geom., 1 (1967), 331–345.

[4] H. Yanamoto, *Quasi-Sasakian hypersurfaces in almost Hermitian manifolds*, Res. Rep. Nagaoka Tech. Coll., 5 (2) (1969), 148–158.

[5] A. Gray, L. M. Hervella, *The sixteen classes of almost Hermitian manifolds and their linear invariants*, Ann. Mat. Pura Appl., 123 (4) (1980), 35–58.

[6] M. B. Banaru, *On minimality of a Sasakian hypersurface in a W_3-manifold*, Saitama Math. J. 20 (2002), 1–7.

[7] M. B. Banaru, *Sasakian hypersurfaces of six-dimensional Hermitian manifolds of the Cayley algebra*, Sbornik: Mathematics, 194 (8) (2003), 1125–1136.

[8] L.V. Stepanova, M. B. Banaru, G. A. Banaru, *On geometry of QS-hypersurfaces of Kählerian manifolds*, Siberian Electronic Mathematical Reports, 15 (2018), 815–822.

[9] A. Abu-Saleem, M. B. Banaru, G. A. Banaru, L. V. Stepanova, *Quasi-Kählerian manifolds and quasi-Sasakian hypersurfaces axiom*, Bul. Acad. Ştiinţe Repub. Moldova, Mat., 93 (2) (2020), 68–75.

Mathematical foundations of economical theory of Leonid V. Kantorovich

Mihail Banaru and Nikolai Morozov

Smolensk State University, Russia
e-mail: mihail.banaru@yahoo.com, vladimirow.kolya2016@yandex.ru

Prominent Russian mathematician Academician Leonid Vital'evich Kantorovich (1912–1986) is the only Nobel laureate in Economics representing Russian Federation / Soviet Union [1], [2], [3].

Almost all the achievements of L. V. Kantorovich in economical theory had a mathematical basis. Back in 1938, when L. V. Kantorovich was solving a specific problem of loading industrial equipment, a young mathematician found a solution to this problem, but along the way discovered that a wide class of industrial problems lends itself to a clear mathematical formulation (and further solution). This is how the foundations were laid for a fundamentally new direction at the intersection of mathematics and economics — linear programming, which made it possible to quantitatively approach many problems of the economy and solve them by numerical methods.

2. In our communication, we consider the mathematical foundations of the contribution of L. V. Kantorovich in economical theory. At the same time, we do not limit ourselves to the consideration of linear programming (one of the founders of which, as we have already mentioned, this mathematician is), but we will also give some applications from mathematical analysis, computational mathematics, discrete mathematics, etc.

Bibliography

[1] S. S. Kutateladze, V. L. Makarov, J. V. Romanovsky, G. Sh. Rubinstein, *The scientific legacy of L. V. Kantorovich (1912–1986)*, Sib. Zh. Ind. Mat., 4 (2) (2001), 3–17.

[2] S. S. Kutateladze, V. L. Makarov, J. V. Romanovsky, G. Sh. Rubinshtein, *Leonid Vital'evich Kantorovich (1912–1986)*, Sibirsk. Mat. Zh., 43 (1) (2002), 3–8.

[3] S. S. Kutateladze, V. L. Makarov, I. V. Romanovskii, G. Sh. Rubinshtein, *L. V. Kantorovich — mathematician and economist*, Novosibirsk: Siberian Branch of Russ. Acad. Sci., 2003.

D-Structures, Generalized Monoid Rings and Analogues of Amitsur's Theorem

Elena Cojuhari[1], Barry Gardner[2]

[1] *Technical University of Moldova, Moldova,*
[2] *University of Tasmania, Australia*
e-mail: [1]elena.cojuhari@mate.utm.md, [2]Barry.Gardner@utas.edu.au

Skew polynomial rings are an important source of examples. These are rings of polynomials over rings A with identity in which the indeterminate does not commute with the elements of A, but the multiplication is controlled by an endomorphism and/or a derivation (of some kind) of A. The generalized monoid rings $A < G, \sigma >$ associated with D-structures σ [3] are analogous to skew polynomials rings, the multiplication being controlled by a family of self-maps of A labelled by the elements of G. They include as special cases the skew monoid rings in the sense of [1].

Amitsur has shown that for every ring A, the Jacobson radical $J(A[X])$ of the (standard) polynomial ring has the form $I[X]$, where I is a nil ideal of A. Much work has been done on analogues of the Amitsur theorem for various kinds of skew polynomial rings. For a survey see the introduction to [4]. We note also that the question "if R is nil, must $R[X]$ be quasiregular (i.e. a Jacobson radical ring)?" is equivalent to the Koethe Problem. In general there is interest in connections between the nil radical of the coefficient ring and the Jacobson radical of its skew polynomial rings.

Here we take first steps in the investigation of the corresponding problems for generalized monoid rings defined by D-structures. The direct analogue of Amitsur's Theorem would be the assertion that $J(A < G, \sigma >) = I < G, \sigma >$ for some nil ideal I of A. However, $I < G, \sigma >$ is only defined if I is invariant under all maps of σ. On the other hand, we have an example (used for another purpose in [2]) of a ring A, a monoid G and a D-structure σ, such that the nil radical $N(A)$ of A is non-zero, but the ideal of $A < G, \sigma >$ generated by $N(A)$ contains an idempotent and so is far from quasiregular. In this case $N(A)$ is not invariant. This investigation is continuing.

Bibliography

[1] G. Abrams and C. Menini, *Skew semigroup rings*, Beitraege Alg. Geom, 37 (1996), no. 2, 209-230.

[2] E. P. Cojuhari and B. J. Gardner, *Radicals and generalizations of derivations*, Bul. Acad. Stiinte Repub. Moldova Mat. No2 (84) 2017, 54-65.

[3] E. P. Cojuhari and B. J. Gardner, *Skew ring extensions and generalized monoid rings*, Acta Math. Hungar. 154 (2018), no. 2, 343–361.

[4] A. Smoktunowicz, *How far can we go with Amitsur's Theorem in differential polynomial rings*, Israel J. Math., 219 (2017), 555-608.

[5] S. A. Amitsur, *Radicals of polynomial rings*, Canadian Journal of Mathematics 8 (1956), 355–361.

[6] E. P. Cojuhari and B. J. Gardner, *Radicals and generalizations of derivations*, ICOR 2015 International Conference on Radicals, Rings, Nearrings and other Algebraic Structures, July 2015, Vorau, Austria.

[7] B. J. Gardner and R. Wiegandt, *Radical theory of rings*, Marcel Dekker, New York and Basel, 2004.

Spectral spaces and spectral compactificationst

Laurenţiu Calmuţchi, Dorin Pavel

UST/UPSC, Chisinau, Republic of Moldova
e-mail: lcalmutchi@gmail.com, paveldorin@gmail.com

,

In there are no concrete indications, then the space is considered a T_0-space.

For every topology T on a set X denote by hT the topology generated by the subspace $T \cup \{X \setminus U\} : U$ is an open compact subset of (X, T). Let hX be the space X endowed with the topology hT. The topology hT is called the Hochster modification of the topology T.

A space X is a spectral space if X contains a open base of open compact subsets and the space hX is compact [2].

Let $Y \subset X$. The set Y is $h-$dence in X, if Y is dence in X in relation to Hochster modification topology onto X.

Theorem 1. *Every spectral T_1-space is Hausdorff space.*

Theorem 2. *The class S of all spectral spaces is a double compactness [1].*

The following statements are correct:

1. *For every space X there exists a unique spectral compactness (sX, s_X) where the space X is $h-$dense and $s_X(x) = x$ for all $x \in X$;*

2. *For every space X there exists the maximal compactification (sX, s_X);*

3. *For every continuous mapping $f : X \to Y$ there exists a unique continuous extension $sf : sX \to sY$ such that the mapping $sf : h_sX \to h_sY$ is continuous too.*

Bibliography

[1] Calmuţchi L.I. *Algebraic and functional methods in the theory of extensions of topological spaces.* Piteşti, Pământul, 2007

[2] Stone, M.H. *Topological representations of distributive lattices and Brouwerian logics*, Čas. Math. Fyz., **67**(1938), 1-25.

Constructing one Symmetric Algorithm based on the Latin Groupoid Isotopes

Liubomir Chiriac, Aurel Danilov, Violeta Bogdanova

Tiraspol State University/State Pedagogical University of Chisinau, Chişinău, Republic of Moldova
e-mail: llchiriac@gmail.com, aureliu.danilov@gmail.com, bogdanovaleta@gmail.com

Dedicated to the memory of Professor Alexandru Basarab

This paper aims to propose a new encryption and decryption algorithm to improve the secure level using, the concept of the Latin groupoid and notion of isotopes.

Encryption is a process which transforms (converts) the original information (readable message) into an unrecognizable form. This new form of the message is entirely different from the original message. Decryption is the process of converting an encrypted message back to its original (readable) format. The original message is called the plaintext message. The encrypted message is called the ciphertext message. Symmetric-key encryption are algorithms which use the same cryptographic keys for both encryption of plaintext and decryption of ciphertext.

In the following we will refer only to symmetric algorithms. Secret key encryption algorithms are characterized by the fact that they use the same key both in the encryption and decryption processes.

Evidently, the key characterizing these algorithms must be known only by the sender and recipient.

Symmetric encryption has the advantage of the following three basic characteristics: 1) the speed with which the message is encrypted and decrypted; 2) ensuring the security of messages; 3) the possibility to encrypt a huge volume of data. The central disadvantage is that the secret key is shared and must be known by both the one who encrypts and the one who decrypts.

We have proposed a simply and efficient Encryption and Decryption Algorithm based on the Latin groupoid and the concept of isotopy. We recall some fundamental definitions and notations [1-10].

Let $(G, \star)$, $(H, \circ)$ be groupoids. An isotopy from $(G, \star)$ to $(H, \circ)$ is an ordered triple: $\phi = (f, g, h)$, of bijections from $(G, \star)$ to $(H, \circ)$, such that $f(a) \circ g(b) = h(a \star b)$ or $h^{-1}(f(a) \circ g(b)) = a \star b$ for all $a, b \in G$.

An $(H, \circ)$ is called an isotope of $(G, \star)$, or $(H, \circ)$ is isotopic to $(G, \star)$ if there is an isotopy $\phi = (f, g, h)$:$(G, \star) \to (H, \circ)$.

If $f : G \to H$ is an isomorphism, then $(f, f, f) : G \to H$ is an isotopy. We can write $f = (f, f, f) : G \to H$. If all 3 permutations coincide: $f = g = h$, then isotopy turns into isomorphism.

A *Latin groupoid* of order n is a $n \times n$ array filled with s, distinct symbols (by convention $\{al_1, ..., al_s\}$),where $s \leq n^2$, such that there are symbols which are repeated twice or more times, in rows or columns.

It should be mentioned that a Latin groupoid is a Latin square of order n, is a $n \times n$ array filled with $n = s$ distinct symbols, such that no symbol is repeated twice in any row or column.

Two Latin groupoids are isotopic if each can be turned into the other by permuting the rows and columns. This isotopy relation is an equivalence relation; the equivalence classes are the isotopy classes.

This cryptographic algorithm is safe in the process of the implementation and it is not complicated to develop a program for the developed algorithm. Cryptographic developed algorithm does not consume large amount of CPU time and space during in the process of encryption and decryption.

In this sense, the authors, based on the proposed algorithms, developed a program in the C++ programming language that works quickly and efficiently.

Bibliography

[1] Liubomir Chiriac, Aurel Danilov, Violeta Bogdanova, *Encryption and decryption algorithm based on the Latin groupoid isotopes*, Acta et Commentationes, Exact and Natural Sciences, TSU, ISSN:2537-6284, Volume 2(14), 2022, Pages 117¡96¿131, E-ISSN:2587-3644 DOI:10.36120/2587-3644.v14i2.117-131

[2] A. Atanasiu, *Securitatea informatiei, (Protocoale de securitate)*, vol. 2 Ed. InfoData, Cluj, 2009

[3] P. Barthelemy, R. Rolland, P. Veron, *Cryptographie, Principes et mises en euvre*, Hermes Science, 2012

[4] V. Shcherbacov, *Quasigroup based crypto-algorithms*, January 2012, https://arxiv.org/abs/1201.3016.

[5] L. Chiriac, A. Danilov, V. Bogdanov, *Utilizarea conceptelor din teoria numerelor in elaborarea algoritmilor criptografici asimetrici*, Conferinta stiintifica nationala cu participare internationala, Universitatea de Stat din Tiraspol, Chisinau, 29 - 30 septembrie, 2020, CZU: 378.147:004+51, ISBN 978-9975-76-312-7, p. 239-247.

[6] L. Chiriac, A. Danilov, *Abordari metodice privind aplicarea quasigrupurilor la dezvoltarea unor metode de criptografie*, Acta et Commentationes, Sciences of Education, nr. 4(22), 2020, CZU: 378.147:004, ISSN 1857-0623, DOI: 10.36120/2587-3636.v22i4.7-17, p. 7-17 (cat B).

[7] Aureliu Danilov, Liubomir Chiriac, *Studierea sistemului criptografic asimetric ElGamal*, SIPAMI. International Symposium "Actual Problems of Mathematics and Informatics": dedicated to the 90th birthday of professor Ion Valuta, November 27-28, 2020, UTM, Chisinau, ISBN 978-9975-45-677-7.

[8] L. Chiriac, A. Danilov, *Abordari metodice in studierea sistemului criptografic asimetric Merkle-Hellman*, Acta et commentationes (stiinte ale Educatiei), Nr. 3(25) / 2021, CZU: 37.016:51, ISSN 1857-0623, DOI: 10.36120/2587-3636.v25i3.7-23, p. 7-23 (cat B).

[9] Liubomir Chiriac, Aurel Danilov. *Aspecte didactice privind studierea algoritmului de criptare RSA, functiilor hash si semnaturii digitale.* CAIM-2022, Proceedings of the 29th Conference on Applied and Industrial Mathematics, Education CAIM 2022, Chisinau, Moldova, August 25-27, 2022, Chisinau, Moldova, dedicated to the memory of Academician Mitrofan M. Choban, Communications in Education, CZU: 37.016:51+004, ISBN 978-9975-76-401-8, p. 125-134.

[10] Liubomir Chiriac, Aurel Danilov, *Metodologia implementarii izotopiilor de grupoizi la criptarea/decriptarea textelor*, A doua Conferinta stiintifica Internationala "Abordari inter/transdisplinare in predarea stiintelor reale, (Concept STEAM)", Chisinau, Republic of Moldova 28 - 29 octombrie, 2022. CZU: 512+004:378.147

The parametric bases in intermediates logics

Ion Cucu

Moldova State University, Chişinău, Republic of Moldova
e-mail: `cucuion2012@gmail.com`

The intermediates logics are intermediary between classical logic and intuitionistic one. A superintuitionistic logic is said to be chain logic if the formula $((p \supset q) \vee (p \supset q))$ is true in it.

Let us to consider the pseudo-Boolean algebra $\langle M; \Omega\rangle$, were $\Omega = \{\&, \vee, \supset, \neg\}$; here $\supset$ is relative pseudocompliment and $\neg$ is pseudocompliment. We say that the function f of algebra $\mathfrak{A}$ can be parametrically expressed via a system of functions Σ of $\mathfrak{A}$, if there exists the functions $g_1, h_1, \ldots, g_r, h_r$ which are expressed explicitly via Σ using superposition such the predicate $f(x_1, \ldots, x_n) = x_{n+1}$ is equivalent to the predicate $\exists\, t_1 \ldots \exists\, t_l \big((g_r = h_1) \& \ldots \& (g_r = h_r)\big)$ on the algebra $\mathfrak{A}$. The system Σ of pseudo-Boolean terms on the set of variables X is parametrically complete in $\langle M; \Omega\rangle$, if we can parametrically express the operations from Ω via functions expressed by terms over Σ.

Let us examine the 5-valued pseudo-Boolean algebra

$$Z_5 = \big\langle \{0, \rho, \tau, w, 1\};\, \&,\, \vee,\, \supset,\, \neg \big\rangle;$$

where ρ and τ are incomparable elements and $0 < w < \rho < 1$; $0 < w < \tau < 1$.

The algebra $Z_3 = \big\langle \{0, w, 1\};\, \&,\, \vee,\, \supset,\, \neg \big\rangle$ is a subalgebra of Z_5.

The function $f(x_1, x_2, \ldots, x_n)$ and $g(x_1, x_2, \ldots, x_k)$ are called permutable if the identity
$f\Big(g(x_{11}, x_{12}, \ldots, x_{1k}), g(x_{21}, x_{22}, \ldots, x_{2k}), \ldots, g(x_{n1}, x_{n2}, \ldots x_{nk})\Big) =$
$= g\Big(f(x_{11}, x_{21}, \ldots, x_{n1}), f(x_{12}, x_{22}, \ldots, x_{n2}), \ldots, f(x_{1k}, x_{2k}, \ldots, x_{nk})\Big)$ hold.

A system Σ is pseudo-Boolean terms is called a parametrically basis in algebra Z_5 if Σ is parametrically complete in this algebra, but every its proper subsystem is not parametrically complete in the algebra Z_5.

The logic of the algebra Z_m we denote by C_m.

Theorem *The cardinality of every parametrical basis in algebra Z_5 is not more than 7 and for every $k = 1, 2, .., 7$, there exists a parametrical basis in Z_5, the cardinality of which is k.*

Example *The system of formulas*

$$\{0, 1, p\&r, p \vee q\},$$
$$\{0, 1, \neg\neg(p\&q), \neg\neg(p \vee q), \neg\neg p\&(p \vee \neg q), \neg\neg p\&(p \vee q \vee \neg q)\},$$
$$\{0, 1, \neg\neg(p\&q), \neg\neg(p \vee q), \neg\neg p\&(p \vee \neg q), \neg\neg p\&(p \vee q \vee \neg q), (p \supset (q \vee \neg q)) \supset p\},$$

are parametric bases in the chain logics C_2, C_3 and L, respectively, where $L \subseteq C_4$.

Hyperbolic Manifolds Based on Geometry of their Submanifolds

Florin Damian

"Vladimir Andrunachievici" Institute of Mathematics and Computer Science, Moldova State University, Chişinău, Republic of Moldova
e-mail: fl_damian@yahoo.com

We describe geometric methods to build and investigate hyperbolic manifolds with certain required geometric properties. We use these examples and methods of metric reconstruction to obtain non-face-to-face incidence schemes for fundamental polyhedra and, as a result, for building new manifolds and some exotic tilings on universal coverage. The communication will be focused on the transfer of methods of discrete geometry to topology and vice versa.

Matrix majorizations and their applications

Alexander Guterman
joint work with Pavel Shteyner and Geir Dahl

Bar-Ilan University, Israel
e-mail: alexander.guterman@gmail.com

Vector majorization is a classical notion useful in many areas of mathematics and their applications, including economics, genetics, etc. There are many ways to define majorizations for real matrices, generalizing the notion of vector majorization. Different types of matrix majorizations have been applied to different areas of research. For example, directional majorization can be applied in economics, while row-stochastic majorization plays an important role in the theory of statistical experiments. We introduce majorizations for sets or families of matrices and investigate them. Also we investigate several known majorizations on the sets of (0,1) and (-1,0,1) matrices. The motivation to study these concepts comes from mathematical statistics and involves the information content of experiments. We characterize so-called minimal cover classes with respect to the weak matrix majorization. We also obtain the complete characterization for the matrix mappings preserving and converting several types of majorizations.

Finite Frobenius groups and maximal one-sided S-systems of orthogonal operations

Kuznetsov Eugene

Moldova State University, Institute of Mathematics and Computer Science, Chisinau, Republic of Moldova
e-mail: kuznet1964@mail.ru

The definition of Frobenius group is well-known in the permutation group theory.

Definition 1. *The transitive irregular permutation group G acting on a set E is called a* ***Frobenius group****, if $St_{ab}(G) = \{id\}$ for any $a, b \in E, a \neq b$.*

In 1902 Frobenius proved the following theorem.

Theorem 1. *A set T of all fixed-point-free permutations in a finite Frobeniusgroup G with the identity permutation id forms a transitive normal subgroup in G.*

Let us note: $T^* = T \cup \{id\}$, $card(T^*) = k$, $card(St_a(G)) = m$, where $a \in E$.

In [1] is shown that over any finite Frobenius group it can be constructed an one-sided S-system of k mutually orthogonal binary operations $A_i(x, y)$, $i = 0, 1, ..., k-1$; moreover, operations $A_2(x, y)$, $A_3(x, y), ..., A_{k-1}(x, y)$ are idempotent quasigroups.

A following theorem holds.

Theorem 2. *A set T^* is a group transversal in G to $H = St_a(G)$, $a \in E$. Corresponding transversal operation $\left(E, \overset{(T)}{\bullet}\right)$ is mutually orthogonal to any operation $A_i(x, y)$, $i = 0, 1, ..., k-1$. So we obtain a system of $k+1$ mutually orthogonal binary operations (where $k-1$ operations are quasigroups).*

Bibliography

[1] Kuznetsov E, *Frobenius groups and one-sided S-systems*, Quasigroups and related systems, **3**(1996), p. 21-40.

A projective plane as an incidence system of cosets in the sharply 2-transitive permutation loop to a suitable subloop

Kuznetsov Eugene

Moldova State University, Institute of Mathematics and Computer Science, Chisinau, Republic of Moldova
e-mail: kuznet1964@mail.ru

The so-called incidence systems are often used in finite geometries.It may be constructed over different algebraic systems - over algebras, groups, quasigroups, loops etc.

Author in [1] introduced the following notion.

Definition 1. *An incidence system $\Sigma(G)$ of left (right) cosets over the system Σ of some subgroups of a group G is an incidence system (A, L, I) such that:*

1. *"Points" from A are all elements of the group G.*
2. *"Lines" from L are left (right) cosets in G to the subgroups from Σ.*
3. *An incidence relation is a belonging relation.*

Also in [1] the following theorem was proved.

Theorem 1. *Let G be a group and Σ be a such system of subgroups in the group G that system $\Sigma(G)$ can be supplemented up to some projective plane π. Then the following statements are true:*

1. *Group G is isomorphic to a sharply 2-transitive permutation group on some set E.*
2. *Plane π is a plane dual to the translation plane.*

The Definition 1 may be generalized up to a case of incidence system over loop.

Definition 2. *An incidence system $\Sigma(G)$ of left (right) cosets over the system Σ of some suitable subsets of a loop G is an incidence system (A, L, I) such that:*

1. *"Points" from A are all elements of the loop G.*
2. *"Lines" from L are left (right) cosets in G to the subsets from Σ.*
3. *An incidence relation is a belonging relation.*

Then the following two theorems hold.

Theorem 2. *Let G be a sharply 2-transitive permutation loop on a finite set E. Then there exist a system Σ of suitable subsets of the loop G such that the corresponding incidence system $\Sigma(G)$ of left (right) cosets over the system Σ can be univocal supplemented up to some projective plane π.*

Theorem 3. *Any finite projective plane π may be univocally obtained from an incidence system $\Sigma(G)$ of left (right) cosets over the system Σ of suitable subsets of some sharply 2-transitive permutation loop G on a finite set E.*

Bibliography

[1] Kuznetsov E, *Incidence systems over groups that can be supplemented up to projective planes*, *Quasigroups and related systems*, **5**(1998), p. 35-52.

A new intersection-graph type for modules

Mamoon Ahmed

Princess Sumaya University, Amman, Jordan
e-mail: m.ahmed@psut.edu.jo

Let R be a ring with unity and M a left R–module. We inves- tigate a new graph associated with M called the simple–intersection graph of M, denoted by $GS_R(M)$. Our focus is on exploring the relationship between different algebraic properties of M and properties $GS_R(M)$, including connectedness, girth and dominating sets. Our re- sults encompass determining the girth and diameter of $GS_R(M)$ and providing insights on the clique and domination numbers.

The Schouten-Van Kampen connection on the tangent bundle endowed with a general natural metric

Simona-Luiza Romaniuc

Universitatea Tehnică "Gh. Asachi", Iași, Romania
e-mail: simona-luiza.romaniuc@academic.tuias

We determine the Schouten–Van Kampen connection associated to the Levi-Civita connection of a general natural metric on the total space TM of the tangent bundle of a Riemannian manifold. We provide the necessary and sufficient conditions for the obtained Schouten–Van Kampen connection to be torsion free and then to coincide with the Levi-Civita connection. We characterize the general natural α-structures on TM, which are parallel with respect to the torsion free Schouten-Van Kampen connection. Finally, we obtain the (para-)Kähler structures of general natural lift type on TM, for which the α-structure is parallel with respect to both Levi-Civita and Schouten-Van Kampen connection.

On recursive differentiability of Bruck-Belousov prolongations of quasigroups

P. Syrbu, E. Cuznețov

Moldova State University, Chişinău, Republic of Moldova
e-mail: parascovia.syrbu@gmail.com, lenkacuznetova95@gmail.com

A prolongation of a finite quasigroup Q of order q is a method of construction of a quasigroup Q' of order $q+k$, where $1 \leq k \leq q$, using Q. R.H. Bruck proposed a construction of such prolongations for idempotent quasigroups (see [1]). V. Belousov considered this method for an arbitrary transversal of the corresponding Latin square (see, [2]). Other ideas of finite quasigroups prolongations, have been proposed by G. Belyavskaya, I. Deriyenko, W. A. Dudek, and V. Shcherbacov.

The recursive 1-differentiability of Bruck-Belousov prolongations of quasigroups is studied in the present work. A binary quasigroup is called recursively 1-differentiable if the groupoid $(Q, \circ)$, where $x \circ y = y \cdot (x \cdot y)$, $\forall x, y \in Q$, is a quasigroup. At present it is known that there exist recursively 1-differentiable finite binary quasigroups of order q, for every q excepting $1, 2, 6$, and possibly $14, 18, 26$ (see, [3]). Another open problem is to find the maximum order r such that a finite binary (or n-ary) quasigroup is recursively r-differentiable (see, [4]). We give necessary and sufficient conditions when the Bruck-Belousov prolongations of quasigroups are recursively 1-differentiable. Also we study the recursive differentiability of finite n-ary quasigroups ($n \geq 2$.) In particular, it is shown that there exist finite recursively 1-differentiable n-quasigroups of order q, for every odd $q \geq 3$ and every $n \geq 2$.

Projective, inductive limits and products of subcategories

Alina Țurcanu

Technical University of Moldova, Tiraspol State Universityi, Chişinău
e-mail: alina.turcanu@mate.utm.md

Are studied the left cartesian products. It is proved that the left product of two subcategories (one reflective and one coreflective) can be realized as a projective limit, and the right product - as an inductive limit.

Bibliography

[1] Botnaru D., *Aspecte categoriale ale spaţiilor topologice vectoriale. Subcategorii semireflexive*, Chişinău, Ştiinţa, 2023.

[2] Engelking R., *General topology*, Warszawa, 1985.

[3] Ţurcanu A., *The factorization of reflector functors*, Buletinul Institutului Politehnic din Iasi, Romania, 2007, t. LIII (LVII), f.5, p. 377-391.

7. Education

The symbiosis of mathematical competence-digital competence

Andrei Braicov, Alexandru Bibic

"Ion Creangă" State Pedagogical University, Chişinău, Republic of Moldova
e-mail: braicov.andrei@upsc.md, alex_bibik@icloud.com

Mathematical competence and digital competence are key competences, necessary for every person for harmonious inclusion in society and quality professional insertion. The first is a decisive factor for the development of digital competence, contributing to its consolidation through the formation of essential cognitive and metacognitive skills for the effective and responsible use of digital technologies. It facilitates communication and collaboration in digital environments because it allows the use of common representations and symbols to express ideas, data or information. At the same time, digital literacy stimulates the learning and development of mathematical competence by providing varied and attractive tools, resources, methods and contexts for students and teachers. The article presents an analysis of the interactions between the mentioned competencies and highlights the benefits of this interaction not only for the actors of the educational system, but also for any other people.

Keywords: digital competence, mathematical competence, mathematical education.

Bibliography

[1] Braicov, Andrei; Corlat, Sergiu; Globa, Angela, *ICT in enhanced learning: how to use ICT*, Capitol în Monograph / Eds.: Danguole Rutkauskiene, Oleksandr Suk, Daina Gudoniene. Kharkiv: Planeta print, 2017. 309 p. ISBN: 9786177229734, p. 151 – 192 (42 p).

[2] Braicov, Andrei, *Competența de creare a conținuturilor digitale educaționale de către profesorii de matematică*, Proceedings of the 27th Conference on Applied and Industrial Mathematics. Communications in Education. Târgoviște, September 19-22, 2019, p. 22-28. ISBN 978-9975-76-282-3

[3] Braicov, Andrei; Corlat, Sergiu; Veverița, Tatiana, *Despre competența digitală a cadrelor didactice din învățământul profesional tehnic*, Materialele Conferinței cu participare internațională „Învățământul Superior: Tradiții, Valori, Perspective", 1–2 octombrie 2021. Chişinău: UST, ISBN 978-9975-76-360-8. p. 37 – 41.

Cloud solutions for digital communication in mathematics education

Andrei Braicov,Ilona Popovici

"Ion Creangă" State Pedagogical University, Chişinău, Republic of Moldova
e-mail: braicov.andrei@upsc.md, ilonapopovici8@gmail.com

Cloud computing have proven their efficiency in various fields of human activity, including education. This article presents a critical analysis of cloud tools that can be leveraged to enhance digital communication in mathematical education. The correspondence between digital communication competence descriptors and the mentioned tools is highlighted. Cloud solutions for delivering and collaboratively synchronous editing mathematical content remotely are also addressed, streamlining both teacher-learner communication and the entire process of teaching-learning and assessment in mathematics

Keywords: digital communication, mathematical education, cloud computing, competence.

Bibliography

[1] Braicov, Andrei; Popovici, Ilona, *Necesitatea utilizării TIC în formarea competenței de comunicare*, In: Materialele Conferinței Republicane a Cadrelor Didactice Științe exacte. Vol. 1, 10-11 martie 2018, Chișinău. Chișinău, Republica Moldova: Universitatea de Stat din Tiraspol, 2018, pp. 268-273. ISBN 978-9975-76-229-8.

[2] Braicov, Andrei; Popovici, Ilona,*Despre aspectele teoretice ale dezvoltării competenței de comunicare*, In: Acta et Commentationes. Științe ale Educației. 2020, nr. 1(19), pp. 31-41. ISSN 1857-0623. DOI: https://doi.org/10.36120/2587-3636.v19i1.31-41

[3] Braicov, Andrei; Popovici, Ilona; Viscu, Irina,*Utilizarea soluțiilor cloud pentru promovarea metodelor activ-participative și dezvoltarea competenței de comunicare*, In: Probleme actuale ale didacticii științelor reale consacrată aniversării a 80-a a profesorului universitar Ilie Lupu. Ediția a II-a Vol.1, 11-12 mai 2018, Chișinău. UST, 2018, pp. 131-135. ISBN 978-9975-76-238-0.

[4] Braicov, Andrei, *Competența de creare a conținuturilor digitale educaționale de către profesorii de matematică*, Proceedings of the 27th Conference on Applied and Industrial Mathematics. Communications in Education. Targoviste, September 19-22, 2019, p. 22-28. ISBN 978-9975-76-282-3

[5] Popovici, Ilona,*Instrumentarul de implementare a unui model pedagogic de dezvoltare a competenței de comunicare digitală prin utilizarea tehnologiilor cloud*, În: Acta et Commentationes, nr. 3 (25). Chișinău: UST, 2021, pp. 106-120.

[6] Popovici, Ilona,*Demersuri experimentale pentru identificarea dinamicii dezvoltării competenței de comunicare digitală prin tehnologii cloud*, În: Acta et Commentationes, Sciences of Education, nr. 1(27). Chișinău: UST, 2022. P. 128-141. ISSN 1857-0623.

[7] ***, https://www.researchgate.net/publication/326223004_The_Systems_of_Computer_Mathematics_in_th_Cloud-Based_Learning_Environment_of_Educational_Institutions

[8] Popovici, I.,*Metode și tehnici de dezvoltare a competenței de comunicare digitală*, Revistă de teorie şi practică educațională a Centrului Educațional PRO DIDACTICA Nr. 2-3 (138-139), 2023

How to use `ChatGPT` in the teaching-learning process of computer science courses

Ala Gasnaș, Angela Globa

"Ion Creangă" State Pedagogical University,
Chişinău, Republic of Moldova
e-mail: `gasnas.ala@upsc.md, globa.angela@upsc.md`

The use of artificial intelligence in the field of education is a very current topic. `ChatGPT` is an AI tool that offers a number of benefits, including increased student engagement, collaboration and accessibility. `ChatGPT` can be considered an attractive alternative in learning, but at the same time it is necessary to maintain a critical approach in using it and verifying the information provided. This paper examines the opportunities and challenges of using `ChatGPT` in higher education and discusses the potential risks and rewards of these tools.

Keywords: artificial intelligence, Turing machine, didactic process, academic performance.

Diversification of software tools of social statistics with JAMOVIE

Maria Pavel, Dorin Pavel

"Ion Creangă" State Pedagogical University,
Chişinău, Republic of Moldova
e-mail: pavel.maria@upsc.md, pavel.dorin@upsc.md

University teaching staff are put in a position to periodically update the curricular contents of the courses taught, especially those in the field of information technologies, in accordance with modern pedagogical and technological requirements. In the paper, it is proposed to update the statistical data analysis courses by capitalizing on the tools of the jamovi software, which is modern, fast, free, interactive and allows the realization of both descriptive and inferential statistics. There are also concrete examples of obtaining descriptive statistical indicators, as well as examples of performing parametric and non-parametric statistical tests within the jamovi software.

Keywords: scientific research, statistical analysis, statistical software, parametric statistical test, non-parametric statistical test.

Bibliography

[1] Pavel, Maria, Pavel, Dorin,*Metodologia cercetării în domeniul tehnologiilor informaționale în educație* In: Acta et commentationes. Ştiinţe ale Educaţiei. 2023, nr. 2(32), pp. 81-92. ISSN 1857-0623. DOI: https://doi.org/10.36120/2587-3636.v32i2.81-92

[2] ***, https://www.g2.com/categories/statistical-analysis

[3] ***, https://www.r-project.org/about.html

[4] ***, https://blog.efpsa.org/2015/09/01/introducing-jasp-a-free-and-intuitive-statistics-software-that-might-finally-replace-spss/

[5] ***, https://blog.efpsa.org/2017/03/23/introducing-jamovi-free-and-open-statistical-software-combining-ease-of-use-with-the-power-of-r/

[6] ***, https://www.jamovi.org/about.html

[7] Balercă, V., *Particularitățile psihopedagogice de dezvoltare a gândirii critice la elevii claselor primare*, Teză de doctor în științe ale educației. Chișinău: USM, 2023.

[8] Pavel, Dorin, Globa, Angela, Pavel, Maria, *Soluție software gratuită pentru prelucrarea statistică a rezultatelor experimentului psihopedagogic*, In: Conference on Applied and Industrial Mathematics CAIM 2022. Ediţia a 29 (R), 25-27 august 2022, Chişinău. Chișinău, Republica Moldova: Bons Offices, 2022, pp. 174-183. ISBN 978-9975-76-401-8. https://ibn.idsi.md/vizualizare_articol/164184

[9] Pavel, Maria, *Prelucrarea statistică a informației psihopedagogice*, In: Probleme actuale ale didacticii ştiinţelor reale consacrată aniversării a 80-a a profesorului universitar Ilie Lupu. Ediția a II-a Vol.1, 11-12 mai 2018, Chişinău. Chișinău, Republica Moldova: Universitatea de Stat din Tiraspol, 2018, pp. 199-202. ISBN 978-9975-76-238-0. https://ibn.idsi.md/vizualizare_articol/94659

[10] Labăr, Adrian-Vicențiu, *SPSS pentru științele educației*, Iași: Polirom, 2008. 347 p. ISBN 978-973-46-1148-5

Some aspects of bilingual education in the technical higher school in the context of european integration

Nataliia Snizhko

Department of Higher Mathematics,
National University "Zaporizhzhia Polytechnic", Ukraine
e-mail: snizhko.nataliia@gmail.com

Abstract. The work considers the peculiarities of bilingual education at a technical university as one of the components of European integration processes in modern education. It is certain that globalization in the field of international business and scientific cooperation, Ukrainian universities' integration into the international scientific and educational space result in training specialists to know the official languages of business and scientific communication, including English. Therefore, basic courses are taught in English at many universities. The need to create subject-oriented didactic models in which a foreign language acts as a means of studying various subject areas is emphasized. The choice of mathematics as such a field is justified. The author covers issues related to teaching higher mathematics in English for Ukrainian-speaking students of engineering and technical specialties. It also outlines the most common specific problems teachers of higher mathematics face when teaching English, as well as possible ways to overcome these problems. Special attention is paid to differences in approaches to teaching higher mathematics in Ukraine and abroad. The need to create appropriate methodical manuals, which would combine the principles of Ukrainian and European mathematics education, is emphasized. The work also considers the specifics of the English mathematical language (since there is a need to write articles, annotations, abstracts, as well as communication with English-speaking colleagues). The author also draws attention to the benefits of learning English and the prospects that open up to students in this regard in the context of European integration processesi.

Keyords: higher mathematics, foreign language education, bilingual education, model of bilingual education, subject-oriented didactic models, academic mobility, mathematical terms, communication, glossary.

Improving education experience with Augmented Reality (AR): Exploring the fascinating world of Pi

Titchiev Inga[1,2], Caftanatov Olesea[1], Iamandi Veronica[1], Talambuta Dan[1]

"Vladimir Andrunachievici" Institute of Mathematics and Computer Science,
State University of Moldova
"Ion Creangă" State Pedagogical University,
Chişinău, Republic of Moldova
e-mail: inga.titchiev@math.md, olesea.caftanatov@math.md, veronica.gisca@gmail.com, dantalambuta@gmail.com

We live in the era when information is everywhere, and it is more important that it be relevant and presented in a way that is easily perceived and interactive to be understood. Augmented reality (AR) techniques offer us this possibility.

The educational application[1] of augmented reality delivered via mobile device that engages pupils with a wide range of multi-sensory learning experiences, could potentially provide rich, contextualized learning for understanding concepts related to transcendental number Pi.

Therefore, this research explores the development of AR scenarios that leverage the intriguing properties of the mathematical constant Pi to enhance mathematics learning for students with diverse learning styles [2]. Through tailored AR experiences, visual, auditory, and kinesthetic learners embark on an immersive journey, delving into the mysteries of Pi and its significance in various mathematical concepts. The scenarios encompass interactive visualizations, musical compositions [3], and hands-on explorations, catering to the unique preferences and needs of each learning style.

Changing the abstract concept to the tangible one, allows pupils not only to see, but also to experience and practice, which takes the education process to a new level. This application can also serve as software assistive technologies for hearing and visually impaired children.

Acknowledgments. *20.80009.5007.22, Intelligent Information systems for solving ill structured problems, knowledge and Big Data processing project has supported part of this research.*

Bibliography

[1] M. Kesim; Y. Ozarslan, *Augmented Reality in Education: Current Technologies and the Potential for Education*, Procedia–Social and Behavioral Sciences 47(810):297–302, 2012.

[2] D. Kolb, *The Kolb Learning Style Inventory*, Version 3, Boston: Hay Group, 1999.

[3] ThePiano.SG, "Musical visualisations of Pi," [Online]. Available: https://www.thepiano.sg/piano/read/musical-visualisations-pi, Accessed on: May 2023.

The benefits and challenges of artificial intelligence in education

Teodora Vascan

"Ion Creangă" State Pedagogical University,
Chişinău, Republic of Moldova
e-mail: teodora_vascan@mail.ru

Students and teachers are already benefiting from AI in their daily lives, in many cases without being aware of its presence. Indeed, AI is already being used to learn foreign languages or perform differentiated tasks for personalized teaching and learning. As we can see, AI has great potential in education, but it lacks in-depth impact analysis and could raise ethical considerations. The article identifies concrete examples and use cases of AI in education.

Index

Index