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# e-Book of Abstracts

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# e-Book of Abstracts

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# Plenary talks

## Algebraic solutions and integrability of planar polynomial differential systems

Dumitru Cozma

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We consider a planar system of differential equations

$$\frac{dx}{dt} = P(x, y), \ \frac{dy}{dt} = Q(x, y), \tag{1}$$

where $P(x, y)$, $Q(x, y)$ are coprime polynomials in $(x, y) \in \mathbb{R}^2$ with real coefficients and $n = \max\{deg\, P, deg\, Q\}$ denotes the degree of the polynomial system.

In this paper we investigate the still open problem for planar systems of differential equations (1): *under which conditions do the original and the linearized systems have the same qualitative behavior and the same topological structure around a singular point* $(x_0, y_0)$ ? This problem has been solved for (1) unless if the singular point $(x_0, y_0)$ is of a center or a focus type and $n \geq 3$.

We consider the problem of distinguishing between a center and a focus for system (1) with algebraic solutions which is equivalent to the problem of local integrability of (1) in the neighborhood of a singular point $(x_0, y_0)$. This kind of integrability is called Darboux integrability and it provides a link between the integrability of polynomial differential systems and the number of invariant algebraic curves they have. We study the following problems:

(i) Find the subclass of systems (1) which has a given number $M$ of invariant algebraic curves of respective degrees $d_1, d_2, ..., d_M$.

(ii) For this subclass find the integrability conditions such that the singular point $(x_0, y_0)$ is a center.

We discuss the difficulty of these problems and present the results concerning the relation between the existence of invariant algebraic curves, the Liapunov quantities and the integrability of polynomial differential systems. As applications we consider several families of cubic differential systems ($n = 3$) with a singular point of a center or a focus type and show that the existence of invariant algebraic curves influences the number of local limit cycles and the number of algebraic limit cycles.

## Control in minimum time for phase-field equations

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The minimum time optimal control problem resides in searching for a constrained internal control, active only from a starting time point to an ending time point and driving the solution of the controlled system from an initial state set to a target set such that the length of the interval where the control is active is the shortest among all such candidates. We present the existence of the minimal time optimal control for phase-field models and provide the maximum principle, that is the first-order necessary conditions of optimality. An application is done for the Allen-Cahn equation.

## Mathematics of phase separation in interacting ternary mixtures under evaporation: The unexpected story of a non-local evolution system

Adrian Muntean

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We study a nonlinear coupled parabolic system with non-local drift terms modeling at the continuum level the inter-species interaction within a ternary mixture that allows the evaporation of one of the species. In the absence of evaporation, the proposed system coincides with the hydrodynamic limit of a stochastic interacting particle system of Blume–Capel–type driven by the Kawasaki dynamics. Similar governing dynamics are found in models used to study morphology formation in the design of organic solar cells, thin adhesive bands, and other applications. We investigate the well-posedness of the target system and present preliminary numerical simulations which incorporate 'from the top' evaporation into the model. We employ a finite volumes scheme to construct approximations of the weak solution and illustrate how the evaporation process can affect the shape and connectivity of the evolving-in-time morphologies.

*This is a report on recent joint work with Rainey Lyons (University of Colorado Boulder, USA), Andrea Muntean (Karlstad University, Sweden), and Emilio N.M. Cirillo (La Sapienza University, Rome, Italy).*

## Design and development of intelligent tourist destination image decision information system

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The present talk is focused on the design and development of an intelligent tourist destination image decision information system. The proposed system reflects the influence of criteria of selection of tourist destination based on destination image features as well as an added advantage of redesigning the tourist destination selection plan as per destination image criteria. The destination image of tourist places is an essential component that influences the thinking phenomenon of tourists regarding the selection of tourist places. The process of selection of tourist places among the various tourist destinations is a vague and imprecise one. Decision-making process becomes more complex as it is influenced by many aspects such as brand awareness, brand attitude, adventure, romantic, spirituality, tourists' satisfaction, tourist loyalty, perceived quality, collectively describe the destination image as significant component. In order to develop the system, we use essential soft computing tools such as Fuzzy sets, Linguistic Variables, Triangular Fuzzy Numbers, Trapezoidal Fuzzy number and etc. The purpose of the proposed Intelligent Information system is to provide the most likely ranking of the selected tourist destinations. This system will act as a referral system among tourists, tour operators, and respective tourism-related administrative bodies. The policy makers and the other tourism related administrative people can make policies for improving the low ranked destinations

**Keywords**: Soft Computing, Tourist destination, Intelligent System.
**AMS Subject Classification**: 68P20,94D05,

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# 1. Partial Differential Equations

# Determination of some solutions of the stationary 3D Navier-Stoke equations

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The following systems of partial differential equations is examin:

$$\begin{cases} \dfrac{P_x}{\mu} + uu_x + vu_y + wu_z = \lambda\Delta u + F_x, \\ \dfrac{P_y}{\mu} + uv_x + vv_z + wv_z = \lambda\Delta v + F_y, \\ \dfrac{P_z}{\mu} + uw_x + vw_y + ww_z = \lambda\Delta u + F_z. \end{cases} \tag{1}$$

$$u_x + v_y + w_z = 0. \tag{2}$$

Here $P = P(x,y,z)$, $u = u(x,y,z)$, $v = v(x,y,z)$; $w = w(x,y,z)$; $F = F(x,y,z)$; $u_x = \dfrac{\partial u}{\partial x}$; $\Delta u = u_{xx} + u_{yy} + u_{zz}$; $x, y, z \in R$.

Systems (1), (2) describes the process of the stationary flow of a liquid or gas in three-dimensional space. The system consists of (1), (2) represents the Navier-Stokes equations in the case of three-dimensional stationary motion of a viscous incompressible fluid or gas.

The $P$ function represent the pressure of the liquid, and $u$, $v$, $w$ functions represent the flow velocity component of the liquid or gas. The function $F$ represents the external force and is potential in nature. The constants $\lambda > 0$ and $\mu > 0$ are a determined parameter of the studied liquid's (or gas) viscosity and density.

We mention here that $\lambda = c/R_e, c > 0$, where $R_e$ is the Reynolds number.

A number of solutions to the stationary equations in the two-dimensional case have been determined in the papers [1], [2].

The following theorem is true:

**Theorem 1.** *Either that in the connected domain $D$ the functions $u(x,y,z)$, $v(x,y,z)$, $w(x,y,z)$ and $P(x,y,z)$ admit continuous partial derivatives up to and including the second order. If in this domain the functions $u, v, w$ and $P$ verify the following equalities:*

$$\begin{aligned} u_y &= v_x, u_z = w_x, w_y = v_z, u_x + v_y + w_z = 0 \\ P &= \mu\left[F + C - 0,5\left(u^2 + v^2 + w^2\right)\right] \end{aligned} \tag{3}$$

These functions are the solutions of systems (1), (2) for any constant $C$.

We will give some examples of solutions of the system 1, 2 determined by the formulas (3).

**Example 1.** For the flow velocity components, we have $u = cxt^{-1,5}$, $v = cyt^{-1,5}$, $w = czt^{-1,5}$; $t = x^2 + y^2 + z^2$, $c$ – constant. The pressure $P$ is determined from (3).

The following theorem generates a series of solutions of equations (1), (2).

**Theorem 2.** *May it be $f = a(x)b(y)c(z)$, where the functions $a(x)$, $b(y)$ and $c(z)$ are doubly differentiable and solutions of the following equations:*
$a^{''} = ra; b^{''} = sb; c^{''} = lc, r + s + l = 0$, *here $r$, $s$, $l$ are constants.*

*Then for the flow velocity components of the problem (1), (2) we have $u = f_x, v = f_y, w = f_z$. The pressure $P$ is determined from (3).*

**Example 2.** The components of velocity are: $u = 3c\sin(5z)e^{3x+4y}$, $v = 4c\sin(5z)e^{3x+4y}$, $w = 5c\cos(5z)e^{3x+4y}$; $c$ – constant. The pressure $P$ is determined from (3).

The following theorem gives us solutions in which the viscosity parameter $\lambda$ participates explicitly.

**Theorem 3.** *May it be $u = C_1 f(t)$, $v = C_2 f(t)$, $w = C_3 f(t)$, $t = ax + by + cz$; where $a, b, c, C_1, C_2, C_3$ are constant and function $f(t)$ admit continuous derivatives up to and including the second order. If the followings conditions are met:*

$$\begin{aligned} &aC_1 + bC_2 + cC_3 = \quad 0; aC_3 - cC_1 = \alpha s; cC_2 - bC_3 = \beta s; \\ &r_1 = \alpha r; r_2 = \beta r, \end{aligned} \tag{4}$$

*where $s = bC_1 - aC_2 \neq 0$; $r = ab(C_2^2 - C_1^2) + C_1C_2(a^2 - b^2) - cC_3(bC_1 - aC_2)$;*
$r_1 = ac(C_1^2 - C_3^2) + C_1C_3(c^2 - a^2) + bC_2(cC_1 - aC_3)$;
$r_2 = bc(C_3^2 - C_2^2) + C_2C_3(b^2 - c^2) + aC_1(bC_3 - cC_2)$.

Then, because $r = 0$, $r_1 = 0$ and $r_2 = 0$, the function $f$ is determined from the equation

$$s\lambda f^{'} = 2kt + m; from\ where f(t) = (kt^2 + mt + h)/\lambda s, \tag{5}$$

$k, m, h$ are arbitrary constants.

We note that the function $f$ exists and has the form (5), if the following conditions hold: $bC_1 \neq aC_2$, $aC_3 \neq cC_1$ and $cC_2 \neq bC_3$.

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## New fixed point properties for a general class of positive operators

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In this paper we concern with a general class of positive operators of discrete type acting on the space of real valued functions defined on a plane domain. Based on the weakly Picard operator and the concentration principle, we study for this class, the good and special weakly Picard operator property.

## On a nonlocal and nonlinear second-order anisotropic reaction-diffusion system with in-homogeneous Cauchy-Neumann boundary conditions. Applications on epidemic infection spread

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In our current paper we are following the results obtained by Pavăl et al. in [3] and study a nonlocal form of the system they propose. First we are performing a qualitative analysis (see [1], [2] and references therein) for the equivalent non-local second-order system of coupled PDEs, equipped with nonlinear anisotropic diffusion and cubic nonlinear reaction. Our PDEs system is also implementing a SEIRD (Susceptible, Exposed, Infected, Recovered, Deceased) epidemic model. In order to be able to compare with the before mentioned results we use the same hypothesis on the input data: $S_0(x)$, $E_0(x)$, $I_0(x)$, $R_0(x)$, $D_0(x)$, $f(t,x)$ and $w_i(t,x), i = 1,2,3,4,5$, and we prove the well-posedness of a classical solution in $C((0,T], C^1(\Omega))$, extending the types already proven by other authors. Secondly we construct the implicit-explicit (IMEX) numerical approximation scheme which allows to compute the solution of the system of coupled PDEs. The results are then compared with the ones obtained by [3].

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# 2. ODEs; Dynamical Systems

## Averaging in a generalized multifrequency system with a delay

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Consider a system of differential equations with linearly transformed arguments

$$\frac{da}{d\tau} = \varepsilon^{\kappa_1} X(\tau, a_\Lambda, \varphi_\Theta), \qquad \frac{d\varphi}{d\tau} = \frac{\omega(\tau)}{\varepsilon^{\kappa}} + \varepsilon^{\kappa_2} Y(\tau, a_\Lambda, \varphi_\Theta), \tag{1}$$

where $\kappa_1 \geq 0$, $\kappa_2 \geq 0$, $\kappa > 0$; $\varepsilon \in (0, \varepsilon_0]$, $\tau \geq 0$, $a \in D \subset \mathbb{R}^n$, $D$ is bounded convex domain, $\varphi \in \mathbb{R}^m$; $a_\Lambda(\tau) = \Big(a(\lambda_1\tau), \ldots, a(\lambda_p\tau)\Big)$, $0 < \lambda_1 < \cdots < \lambda_p \leq 1$, $\varphi_\Theta(\tau) = \Big(\varphi(\theta_1\tau), \ldots, \varphi(\theta_q\tau)\Big)$, $0 < \theta_1 < \cdots < \theta_q \leq 1$.

For the system of equations (1), we construct a system of equations averaged by the components of the fast variables $\varphi_\Theta$

$$\frac{d\overline{a}}{d\tau} = \varepsilon^{\kappa_1} X_0(\tau, \overline{a}_\Lambda), \qquad \frac{d\overline{\varphi}}{d\tau} = \frac{\omega(\tau)}{\varepsilon^{\kappa}} + \varepsilon^{\kappa_2} Y_0(\tau, \overline{a}_\Lambda),$$

where

$$F_0(\tau, a_\Lambda) = \frac{1}{(2\pi)^{mq}} \int_0^{2\pi} \cdots \int_0^{2\pi} F(\tau, a_\Lambda, \varphi_\Theta) d\varphi_\Theta, \quad F = (X, Y), \quad F_0 = (X_0, Y_0).$$

The method of averaging by phase variables is used to study a multifrequency system with linearly transformed arguments [1, 2]. On the contrary to multifrequency systems of a standard form, we consider the systems of equations, in which the speed of amplitude and phase variables can depend on a small parameter to various degrees.

The existence and uniqueness of the solution is proved under the condition of the existence of a unique solution of the averaged problem. An estimation of the averaging method, which clearly depends on small parameters, is found in the paper. Initial and multipoint conditions for solutions are considered.

We will show that on the interval $[0, L\varepsilon^{-\kappa_1}]$ under certain sufficient conditions and for a sufficiently small $\varepsilon^* \leq \varepsilon_0$ for each $\varepsilon \in (0, \varepsilon^*]$ there exists only one solution $(a, \varphi) \in \mathbb{C}^1[\tau, \tau + L]$ for the system (1) and for all $[0, L\varepsilon^{-\kappa_1}] \times (0, \varepsilon^*]$ the estimation

$$\varepsilon^{\kappa_2} \|a(\tau, \varepsilon) - \overline{a}(\tau)\| + \varepsilon^{\kappa_1} \|\varphi(\tau, \varepsilon) - \overline{\varphi}(\tau, \varepsilon)\| \leq c_1 \varepsilon^{\alpha + \kappa_1 + \kappa_2}$$

is true. Here $\alpha = \kappa/(mq)$, $a|_{\tau=0} = \overline{a}|_{\tau=0} = \overline{y}$, $\varphi|_{\tau=0} = \overline{\varphi}|_{\tau=0} = \overline{\psi}$, $c_1 > 0$ and does not depend on $\varepsilon$.

The oscillatory integrals corresponding to the system of equations (1) of the form

$$I_k(\tau, s, \overline{s}, \varepsilon) = \int_t^{t+\tau} f(s, \varepsilon) \exp\Big(\frac{i}{\varepsilon^{\kappa}} \int_{\overline{s}}^{s} \gamma_k(z) dz\Big) ds,$$

$$\gamma_k(\tau) = \sum_{\nu=1}^{q} \Big(k_\nu, \theta_\nu \omega(\theta_\nu \tau)\Big),$$

where $\tau \in [0, L]$, $s, \overline{s} \in \mathbb{R}_+$, $\varepsilon \in (0, \varepsilon_0]$, $k_\nu \in \mathbb{Z}^m$, $k = (k_1, \ldots, k_q) \in \mathbb{Z}^{mq} \setminus \{0\}$ are built to proof the results.

The results are illustrated by example. Consider a single-frequency system of equations

$$\frac{da}{d\tau} = \sqrt[3]{\varepsilon}\big(b_1 + b_2 \cos(k\varphi + l\varphi_\theta)\big), \qquad \frac{d\varphi}{d\tau} = \frac{d_1 + 2d_2\tau}{\sqrt{\varepsilon}},$$

where $b_\nu, d_\nu \in \mathbb{R}$, $b_2 \neq 0$, $d_1, d_2 \neq 0$; $\theta \in (0, 1)$, $k, l \in \mathbb{Z}$, $k + l\theta = 0$; $a(0, \varepsilon) = \overline{y}$, $\varphi(0, \varepsilon) = 0$.

We obtain the estimation $|a(\tau, \varepsilon) - \overline{a}(\tau)| \leq |b_2| e^{|b_1|} \varepsilon^{7/12} + O(\varepsilon^{13/12})$.

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## Local Stability in a SEIR Type Epidemic Model

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A SEIR type epidemiological model, with Holling type II transmission and removal rates, is investigated. The model is reduced to a 3D dynamical system, described by a system of three nonlinear ordinary differential equations, depending on seven positive parameters. Local dynamics and bifurcations of this model are investigated, with emphasize on parameter strata near the hypersurface $\mathcal{R}_0 = 1$. The existence of unique or coexisting attractors, either equilibria or limit cycles, it is proved. Several transitions scenarios are found and emphasized numerically.

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## A system with six parameters related to the competition relationship of two populations

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A biological mathematical model with six parameters, which described a competition relationship between two populations is analyzed. In this paper we study the reduced systems to systems with three positive parameters. The nature of the equilibrium points are analysed. The corresponding global dynamic bifurcation diagram is carried out.

## Facultative mutualisms and $\theta$-logistic growth: how larger exponents promote global stability of co-existence equilibria

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We investigate the stability of co-existence equilibria for 2-species models of facultative mutualism for which birth and death are modeled as separate processes, with possibly distinct types of density dependence, and the mutualistic contributions are either linear or saturating. To provide an unifying perspective, we first introduce and discuss a generic stability framework, finding sufficient stability conditions expressed in terms of reproductive numbers computed at high population densities. To this purpose, an approach based on the theory of monotone dynamical systems is employed. The outcomes of the generic stability framework are then used to characterize the dynamics of the 2-species models models of concern, delineating between decelerating (lower-powered) and accelerating (higher-powered) density dependences. It is subsequently seen that accelerating density dependences promote the stability of co-existence equilibria, while decelerating density dependences either completely destabilize the system via promoting the unboundedness of solutions or create multiple co-existence equilibria.

## On LMIs mathematical context and algorithms for constructing efficient control Lyapunov functions for dynamical systems

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Construction of a Control Lyapunov Function (CLF) for a nonlinear dynamical system is generally a difficult problem, but if a CLF is found, stabilization of the system is straight-forward.

The present paper focuses on the efficiency of using linear matrix inequalities (LMIs) methods and convex programming in process control applications. Models arising from excitable media are taken into account. Starting with the 2d case for such models is convenient in order to test the methods and calculus. The analysis would be useful in further analysis, together with the extension to the 3d models.

**AMS Subject Classification (2010):** 93D30, 93D25, 49J35, 49K35, 93C15, 65K05 **Key words**: dynamical system, control Lyapunov function, optimization, control, convex programming

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## Stability conditions of unperturbed motion for critical three-dimensional differential system of Lyapunov-Darboux type $s^3(1,5)$

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We examine the three-dimensional differential system $s^3(1,5)$ of unperturbed motion of the form

$$\frac{dx^j}{dt} = a^j_\alpha x^\alpha + a^j_{\alpha\beta\gamma\delta\mu} x^\alpha x^\beta x^\gamma x^\delta x^\mu \quad (j,\alpha,\beta,\gamma,\delta,\mu = \overline{1,3}), \tag{1}$$

where $a^j_{\alpha\beta\gamma\delta\mu}$ is a symmetric tensors in the lower indices, by which a total convolution is carried out here. Suppose that for the system (1), the center-affine invariant conditions

$$L_{1,3} > 0, \quad L_{2,3} > 0, \quad L_{3,3} = 0, \tag{2}$$

are satisfied, where

$$L_{1,3} = -I_{1,3}, \quad L_{2,3} = \frac{1}{2}(I^2_{1,3} - I_{2,3}), \quad L_{3,3} = \frac{1}{6}(-I^3_{1,3} + 3I_{1,3}I_{2,3} - 2I_{3,3}),$$

and

$$I_{1,3} = a^\alpha_\alpha, \quad I_{2,3} = a^\alpha_\beta a^\beta_\alpha, \quad I_{3,3} = a^\alpha_\gamma a^\beta_\alpha a^\gamma_\beta.$$

Acording to [2], then by center-affine transformation, the system (1), can be brought to the critical form

$$\frac{dx^1}{dt} = a^1_{\alpha\beta\gamma\delta\mu} x^\alpha x^\beta x^\gamma x^\delta x^\mu, \quad \frac{dx^j}{dt} = a^j_\alpha x^\alpha + a^j_{\alpha\beta\gamma\delta\mu} x^\alpha x^\beta x^\gamma x^\delta x^\mu \quad (j = 2,3; \ \ \alpha,\beta,\gamma,\delta,\mu = \overline{1,3}). \tag{3}$$

In the center-affine condition $\eta = a^{\alpha}_{\beta\gamma\delta\mu\nu}x^{\beta}x^{\gamma}x^{\delta}x^{\mu}x^{\nu}x^{\tau}x^{\xi}\varepsilon_{\alpha\tau\xi} \equiv 0$, the system (3), it is a critical system of Lyapunov-Darboux type, of the form

$$\frac{dx^1}{dt} = 5x^1R(x^1,x^2,x^3), \quad \frac{dx^j}{dt} = a^j_{\alpha}x^{\alpha} + 5x^jR(x^1,x^2,x^3), \quad (j=2,3;\ \ \alpha=\overline{1,3}), \tag{4}$$

where

$$R(x^1,x^2,x^3) = a_1(x^1)^4 + a_2(x^2)^4 + a_3(x^3)^4 + 4a_4(x^1)^3x^2 + 4a_5(x^1)^3x^3 + 4a_6x^1(x^2)^3 + 4a_7x(x^3)^3 + 6a_8(x^1)^2(x^2)^2 + 6a_9(x^1)^2(x^3)^2 + 12a_{10}(x^1)^2x^2x^3 + 12a_{11}x^1(x^2)^2x^3 + 12a_{12}x^1x^2(x^3)^2 + 6a_{13}(x^2)^2(x^3)^2 + 4a_{14}(x^2)^3x^3 + 4a_{15}x^2(x^3)^3.$$

We will introduce the following notation:

$$M = a_1 + 4a_4A_1 + 6a_8A_1^2 + 4a_6A_1^3 + a_2A_1^4 + 12a_{10}A_1B_1 + 12a_{11}A_1^2B_1 + 4a_{14}A_1^3B_1 + 4a_5B_1 + 12a_{12}A_1B_1^2 + 6a_{13}A_1^2B_1^2 + 6a_9B_1^2 + 4a_{15}A_1B_1^3 + 4a_7B_1^3 + a_3B_1^4, \tag{5}$$

where

$$A_1 = (a_3^2a_1^3 - a_1^2a_3^3)L_{2,3}^{-1}, \quad B_1 = (a_1^2a_2^3 - a_2^2a_1^3)L_{2,3}^{-1}.$$

Then, taking into account the Lyapunov Theorem [1, §32] and the expression (5), we have

**Theorem 1.** *The stability of unperturbed motion, described by the critical system* (4)*, is described by one of the following three possible cases*:

**I.** If $M > 0$ *then unperturbed motion is* ***unstable****;*

**II.** If $M < 0$ *then unperturbed motion is* ***stable****;*

**III.** If $M = 0$ *then unperturbed motion is* ***stable****.*

*In the last case, the unperturbed motion belongs to some continuous series of stabilized motions, and moreover this motion is asymptotically stable. The expression $M$ is from* (5)*.*

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## Application of a generalization of the Grey Lotka-Volterra model to an economic model

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A generalized grey Lotka-Volterra model with a finite number of variables is derived by applying the grey modelling method to estimate the parameters of a finite dimensional quadratic Lotka-Volterra system. The model is used to analyze the competition and cooperation relationship between several macroeconomic indicators, namely per capita gross domestic product, non-renewable energy consumption, renewable energy consumption, and greenhouse gas emissions, and to obtain short-time forecasting. The data used in the empirical investigation were taken from the Eurostat database and cover the time period 2010-2022.

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## Quadratic systems possessing one invariant parabola and one invariant hyperbola

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Consider the family of real planar polynomial differential systems possessing a finite number of algebraic solutions and a finite number of singularities, finite or infinite. Following [1] we call *configuration of algebraic solutions* of a polynomial differential system the set of algebraic solutions over $\mathbb{C}$ of the system, each one of these curves endowed with its own multiplicity and together with all the real singularities of this system located on these curves, each one of these singularities endowed with its own multiplicity.

In [1-4] two families of quadratic systems possessing invariant conics (hyperbolas or ellipses) are classified according to their configurations of the corresponding invariant curves. In [5] the invariant criteria for the existence of at least one invariant parabola for a quadratic system are determined.

Here we consider the family *QSPH* of quadratic systems possessing a finite number of singularities (finite and infinite) and at least two invariant conics: one parabola and one hyperbola.

Our main goal is to classify the family of systems *QSPH* according to their geometric properties encoded in the configurations of invariant conics (parabolas and hyperbolas) and straight lines.

We prove that there are exactly 40 distinct configurations each one containing at least one invariant parabola and at least one invariant hyperbola. Moreover using the algebraic invariant theory founded by C. Sibirschi [6] we determine necessary and sufficient conditions for the realization of each one of the possible 40 configurations. We remark that the maximum number of invariant affine curves which could have a configuration is five: *a*) one parabola, one hyperbola and 3 lines; *b*) one parabola, two hyperbolas and 2 lines; *c*) two parabolas, one hyperbola and 2 lines.

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# 3. Mathematical Modeling

# Computer Vision Framework for Automatic Vehicle Detection and Information Extraction

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This research work, which is part of a traffic monitoring project, introduces a computer vision framework for multiple vehicle detection and information extraction, which is based on some novel machine/deep learning and mathematical models. The proposed technique detects the vehicles in the traffic video sequences and also extracts their main information: type, trajectory, license plate, numbers and logos.

A transfer learning-based vehicle detection solution is proposed first. A YOLO-based detector is built by modifying a MobileNet-v2 model and using it for high-level feature extraction. Thus created convolutional neural network (CNN) is then trained and validated on a voluminous vehicle database developed by us. A nonlinear partial differential equation (PDE) - based active contour segmentation is then applied to improve the vehicle detection results.

The track of each detected vehicle is then determined applying a tracking by detection (TBD) approach. It performs a multi-scale analysis - based high-level feature extraction on those detections, using a nonlinear diffusion-based scale-space and deep networks, and matches the vehicle instances on successive frames by using their feature vector distance values.

The type of each detected vehicle is next determined by modifying a GoogLeNet classification model and training, validating and testing the obtained CNN on another vehicle dataset. Then, the class of the land vehicle is obtained as the most common label (Car, Bus, Truck, Tram or Bike) in that vehicle track.

A ensemble learning - based license plate detection technique is then proposed here. A cascade classifier-based vehicle plate detection is performed by modeling and training a Haar feature-based boosting classifier on a voluminous dataset prepared by us and applying the trained detector on the traffic video frames. Next, the vehicle numbers on these license plates are extracted by applying some morphological operations to extract the characters and a template matching process on these alfa-numerical characters.

Next, the vehicle logos are located by training, validation and testing a YOLO V5 network, and applying the obtained detector on the vehicles' subimages. The detected logos could also by recognized by using a logo template dataset.

**Keywords:** CNN-based vehicle detection, tracking by detection, multi-scale high-level feature extraction, vehicle classification, cascade classifier, license plate detection, vehicle logo detection.

**Acknowledgments.** *This research work was supported by a grant of the Romanian Academy, GAR-2023, Project Code 19.*

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## Natural convection in a square cavity filled by an EPCM porous medium

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A special category of porous media with variable properties are those formed by encapsulated phase change materials (EPCM). This type of porous media is formed by particles encapsulating a phase change material (e.g. paraffin). During temperature variation, this material undergoes a phase change (e.g. liquid-solid or solid-liquid) and a latent heat is absorbed or released, see Ghalambaz et al [1].In this work, the particle shell encapsulating the phase change material is assumed to be thin and the solid and fluid phases of the saturated porous medium are assumed to be in local thermal equilibrium. The topic of porous media formed by EPCM is new (see, Li and Chen, [2]) and improvement of the existing mathematical models or new mathematical models are required to improve the accuracy of the simulation. In this work, we intend to extend this study to the classical problem of free convection in a square enclosure filled with EPCM porous media.

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## The problem of parameter estimation in a vegetated Saint-Venant model

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In ASTERIX, a software devoted to hydrological processes, the water flow on vegetated soil surfaces is modelled using an extended Saint-Venant system of equations. Since the water-plant and water-soil friction coefficients cannot be directly measured, the problem of estimating them plays a key role for the model calibration. We use an inverse method and some measurements reported in the literature for finding the optimal values of these parameters which allows us to compare the measured with the calculated data and to understand the ability of the theoretical model and of the numerical scheme to offer a reliable solution to the studied hydrological phenomenon. We will also address an important problem in mathematical modelling: the sensitivity analysis of the solution of the model to the variation of the model parameters.

## Algebraic study and stability analysis for the solutions corresponding to first order chemical reactions mechanisms

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When a first-order chemical reaction is modeled mathematically, it always result a first-order linear differential equation. Therefore, when a mechanism formed exclusively by first-order reactions is studied, the corresponding mathematical model consists on a linear first-order ODE system.

The associated matrix for this kind of problems has a very special format that determines the location of its eigenvalues in the complex plane, as well as its algebraic and geometric multiplicities.

From all of the above emerge –at least from a theoretical viewpoint– nine different cases, where only eight of them can occur in real life.

In this work, these eight possible cases are studied, theoretical and practical examples are given and the stability of the solutions is studied.

**Keywords:** First-order reactions, chemical kinetics, eigenvalues, stabiliby.

## The Weak Solution of Electro-Visco-Elastic Material

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In this work, we consider a mathematical model to describe the process of a static contact between a piezoelectric body and an electrically conductive foundation. The behaviour of the material was modelled with a nonlinear electro-viscoelastic constitutive law, the contact was frictionless, and the result was described with the TRESCA condition. First, we give the variational formulation was derived for the problem, proving the existence and uniqueness of a weak solution of the model. The proof was based on arguments of fixed-point theorem.

**Keywords**: Electro-visco-elastic material, frictionless contact; operators, weak solution.

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## Random multiple–fragmentation and flow of particles on a surface

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We investigate stochastic fragmentation processes for particles with spatial position. The mathematical problem models the time evolution of a system of particles which move on an Euclidean surface driven by a given force (e.g., gravitational, fluid interaction, repulsion/attraction), and split in fragments with smaller masses and velocities. We establish a multiple-fragmentation process and we solve the corresponding stochastic integro-differential equation. Finally, we present several numerical simulations of such processes.

*These results are obtained jointly with Lucian Beznea (Bucharest) and Ioan R. Ionescu (Paris).*

## Mathematical instruments involved in water purification of pharmaceutical pollutants

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Removal of pharmaceutical contaminants from water sources has become particularly important in the last decades as their continuous growth and spread over the entire surface of the earth has been demonstrated. Moreover, their hazardous effects have been highlighted in many studies and their discovery is still far to be finalized. Even if, recently, a lot of works deal with this subject, the novelty aspect is by no means exhausted due to the vastness of the problems raised by this type of pollutants. Our current studies in this field conducted to some innovative technological solutions to remove this kind of pollutants from water sources together with design proposals for rehabilitation of existing cleaning and treatment plants. Since these solutions have at their basis either adsorption on activated carbon or ultra- or nanofiltration, phenomena which can be reduced at infiltration through porous media under special conditions, this work is focused on the modeling of this type of problem. This work details adequate mathematical instruments for the mentioned phenomena. Regarding the choice of decontamination method, an ANOVA multiple-factor is recommended without or with interaction and replication, where all the elements that affect the cleaning or treatment efficiency should be involved. But the main mathematical input is regarded to the flow through porous membrane, which plays a crucial role in the understanding of this depollution phenomenon in order to improve it by taking into account the real conditions. Starting from Navier-Stokes equations combined with appropriate boundary conditions, we arrived to some mathematical physics problems for which we propose variational methods to obtain solutions.

# 4. Real, Complex, Functional and Numerical Analysis

## Studies on new univalent operators defined by using Bessel functions of the first kind

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The results follow the line of research in Geometric Function Theory which focuses on introducing and investigating new operators using the theory of differential subordination initiated by S.S. Miller and P.T. Mocanu. Bessel function of the first kind is first investigated concerning univalence conditions, further applying its univalent form for defining a new integral operator. It is proved that this new operator has certain starlikeness properties and it is highlighted that the famous Alexander, Libera, and Bernardi integral operators can be found for specific values of the parameters involved in it's definition. Furthermore, Bessel function of the first kind is used for introducing a fractional calculus integral operator for which geometric properties of starlikeness and convexity are established.

**Keywords:** Bessel function of the first kind, starlike function, convex function, univalent function, integral operator, differential subordination, special functions, fractional integral.

**MSC:** 30C45, 30C80, 33C10.

## A characterization of Almost Grothendieck sets

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In Banach lattice theory, various sets are defined by using disjoint sequences. Almost Grothendieck sets have been introduced in [1] as a disjoint version of Grothendieck sets. This class of sets characterize the weak Grothendieck property, which is first introduced in [2]. In this talk, we give a new characterization of almost Grothendieck sets and obtain some results on almost Grothendieck operators.

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## Wentzell problems with nonstandard growth conditions

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We treat a general class of anisotropic systems with variable exponents and Leray-Lions type operators. For such systems, we provide existence, uniqueness, and the global regularity of the weak solution. These results were obtained in collaboration with Dr. Alejandro Vélez - Santiago (University of Puerto Rico at Río Piedras, San Juan, Puerto Rico) and they can be applied to various types of problems that represent particular cases of our main class of systems.

## On the multiplicative convergence on the tensor product

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In this presentation, we show that the multiplicative order convergence is appropriate on the Fremlin tensor product of two Archimedean $f$-algebras. Also, we give some results on the multiplicative norm convergence on the Fremlin projective tensor product of two Banach lattice $f$-algebras.

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## Strong sandwich results involving the Riemann-Liouville fractional integral of an extended $q$-hypergeometric function

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The classical theories of differential superordination and subordination have been extended to strong differential superordination and respectively, strong differential subordination. The two new theories have progressed well, revealing significant findings when various operators and specific hypergeometric functions have been included in the studies. The research revealed by this work expands the topic of the investigation by incorporating aspects of fractional calculus and quantum calculus. An extended version of $q$-hypergeometric function is introduced to correspond

to the study of functions from the classes that were previously described and that are particularly defined for strong differential superordination and subordination theories. This work defines the Riemann-Liouville fractional integral applied to the extended $q$-hypergeometric function, used to get strong differential subordinations and superordination results. The theorems established for the strong differential superordination and subordination, establish the best subordinants and respectively the best dominants. Interesting corollaries are exposed for certain functions regarded as best subordinant or best dominant due to their particular geometric characteristics. Sandwich-type theorems and consequences conclude the study, stated to connect the outcomes obtained by applying the dual theories.

**Keywords:** Riemann-Liouville fractional integral, extended $q$-confluent hypergeometric function, strong differential subordination, strong differential superordination, best dominant, best subordinant.

**2020 Mathematical Subject Classification:** 330C45, 30A10, 33D05.

## Some inequalities related to the numerical radius of a bounded operator on a Hilbert space

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We will present some new upper bounds for the numerical radius of a bounded operator on a Hilbert space. The particular case of the composition between the adjoint of an operator and some other operator will play a special role. Finally, we present some results related to a quasi-unitary operator.

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## Almost everywhere convergence of Nörlund means of Walsh-Fourier series

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Earlier, the norm convergence of Nörlund means of Walsh-Fourier series in $L_1$ space and in $C_W$ space was discussed in terms of modulus of continuity [3, 4, 5]. The result of Móricz and Siddiqi were generalized to homogeneous Banach spaces and dyadic Hardy spaces [1] as well. In the present talk we mention some new theorem with respect to norm convergence of Nörlund means. The results are related to the corresponding Lebesgue constants of Nörlund means. In the paper [2], we proved two-sides estimates for Lebesgue constants of Nörlund means. We discuss the almost everywhere convergence of Nörlund means under some assumption on the associated Lebesque constants [2].

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## Novel multi-step iterative methods based on homotopy perturbation technique

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This paper presents the development of novel multi-step iterative methods for solving nonlinear equations, utilizing the Homotopy perturbation technique. The proposed methods, free from second derivatives, include a two-step method with fourth-order convergence and a three-step method with eighth-order convergence. These methods combine the Homotopy perturbation method, Haar wavelets quadrature rule, and numerical techniques to improve efficiency and accuracy. Numerical comparisons demonstrate that the new methods outperform existing iterative methods, including Noor's and Chun's methods, in terms of convergence rate and computational efficiency. The results highlight the potential of these methods for various scientific and engineering applications, offering faster and more accurate solutions to nonlinear equations.

## Rhaly operators and statistical convergence, statistical boundedness

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We study Rhaly operators on separable Hilbert spaces. Such operators are defined by terraced matrices, generated by a sequence of complex numbers.

We try to find necessary and sufficient conditions for a terraced matrix to generate a bounded operator, or a compact operator. This leads to statistical convergence and statistical boundedness, namely to study rare subsequences of the natural numbers sequence, and the corresponding subseries of the harmonic series.

## Recent investigations on the univalence of Gaussian hypergeometric function by means of Geometric Function Theory

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In the framework of Geometric Function Theory, the Gaussian hypergeometric function has been studied from multiple angles, including univalence conditions. Such conditions have been obtained using the methods of differential subordination theory in 1990 by S.S. Miller and P.T. Mocanu, considering the parameters related to the Gaussian hypergeometric function as real numbers. Recent studies have extended those results taking the parameters as complex numbers, developing new univalence conditions which are proved to be connected to the initial one established by Miller and Mocanu. The theories of differential subordinations and superordinations are employed for the new outcome, perfected by the method of the subordination chains.

**Keywords:** Gaussian hypergeometric function, starlike function, convex function, differential subordination, differential superordination, subordination chain.

**MSC:** 30C80, 33C15, 30C45.

## A second-order fractional steps type method to approximate a nonlinear reaction-diffusion equation with in-homogeneous Cauchy-Neumann boundary conditions

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The paper concerns with the approximation of solutions to the nonlinear reaction-diffusion equation, endowed with in-homogeneous Cauchy-Neumann boundary conditions. It extends the already studied types of boundary conditions, proven in a previous paper of us and also by other authors, in order to make the problem more suitable for describing important phenomena of two-phase systems. The convergence for a new iterative second order scheme of fractional steps type, associated to the nonlinear parabolic problem is established. With the purpose of simplifying the numerical computation, we formulate a conceptual numerical algorithm, that we intend to implement and compare to other methods of approximation for the problem.

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# Estimates in Banach spaces for weakly singular integral operators and their applications in approximate solving of integral equations

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To apply computational algorithms for the approximate solving of various classes of integral equations with weak singularities efficiently, one has to use certain upper bounds in Banach spaces for the weakly singular integral operators (WSIO) and their modifications occurring in these algorithms.

Note that only the existence of constants in the upper bounds for WSIO was proved (Muskhelishvili, N.I., 1969) however, this does not permit to use computational algorithms in practice efficiently.

In this paper, we compute the above-mentioned constants in the space $C(a,b)$ of continuous functions; the Hölder space $H_\beta(a,b),\ 0<\beta<1$; the Lebesgue space $L_p(a,b),\ 1<p<\infty$ for WSIO and some their modifications that appear in proposed algorithms. Then, based on the obtained constants, we establish invertibility conditions for the modified WSIO in these spaces.

Based on the obtained results, we give theoretical justification in the indicated Banach spaces for the following approximate methods in solving weakly singular integral equations (WSIE) of the second kind:

1) for collocations and spline-collocations algorithms in solving Fredholm and Volterra WSIE of the second kind;

2) for quadratures and spline-quadratures algorithms in solving Fredholm and Volterra WSIE of the second kind.

## 5. Probability Theory, Mathematical Statistics, Operations Research

# Matrix games and parallel algorithms for determining equilibrium profiles in pure strategies

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**Basic sequential algorithm**. We consider the bimatrix game in the following strategic form $\Gamma = \langle I, J, A, B\rangle$ and denote by $NE[\Gamma]$ the set of all equilibrium profiles in the game $\Gamma$. We will use the following sequential algorithm: 1) for any fixed column $j \in J$,the sets $Br_1(j) = Arg\max\limits_{i\in I} a_{ij}$ and $Br_2(i) = Arg\max\limits_{j\in J} b_{ij}$ are determined; 2) the graphs $GrBr_1$, $GrBr_1$ of the point-to-set application $Br_1$ and $Br_2$ are built; 3) construct the set $NE = GrBr_1 \cap GrBr_2$.

**Generation of sets of bimatrix subgames as a result of the division into blocks of global matrices.** We denote by $R$ the set of processes (processors, computing elements) of a parallel computing system and by $I_r$, respectively $J_r$ the lines (columns) of the matrices $A_r$, $B_r$ distributed to the process $r \in R$. For the characterization of the algorithms of division and distribution on matrix-like systems we will introduce the following functions. The functions $\alpha : I \times R \to I_r$ and $\beta : J \times R \to J_r$ will be called Division and Distribution of the Elements of a Matrix (D&DEM function), if they verify the following property: for any $i \in I$ there is a single process $r$ and $i_r \in I_r$, such that $\alpha(r, i) = i_r$, and similarly, for any $j \in J$ there is only one process $r$ and $j_r \in J_r$, so that $\beta(r, j) = j_r$. Also the functions $\varphi : I_r \to I$ and $\psi : J_r \to J$ will be called Restoring the Elements of a Matrix (REM function), if they verify the following property: for any $r \in R$ and $i_r \in I_r$ there is a single element $i \in I$ such that $\varphi(i_r) = i$ and similarly, for any $r \in R$ and $j_r \in J_r$ there is a single element $j \in J$, so that $\psi(j_r) = j$.Thus, using the D&DEM functions we will obtain a series of sub-matrices $\left\{A_r = \left\|a_{\alpha(r,i)\beta(r,j)}\right\|_{i\in I}^{j\in J} \equiv ||a^r_{i_r j_r}||_{i_r\in I_r}^{j_r\in J_r}\right\}_{r\in R}$, and $\left\{B_r = \left\|b_{\alpha(r,i)\beta(r,j)}\right\|_{r\in R} \equiv ||b^r_{i_r j_r}||_{i\in I_r}^{j\in J_r}\right\}_{r\in R}$ which are a result of the division and distribution of matrices $A$ and correspondingly $B$. Using the REM we will rebuild the global matrices $A$ and $B$. If the submatrices have the same size then these submatrices can generate a series of sub-games $\Gamma_r = \langle I_r, J_r, A_r, B_r\rangle$ and denote by $NE[\Gamma_r]$ the set of Nash equilibrium profiles.

**Basic Parallel Algorithm to find Nash equilibrium profiles.** We will present the following parallel algorithm for determining the solutions of matrix games in pure strategies. **A)** Data parallelization: using the D&DEM functions each process $r \in R$ "gets" the pair of matrices $A_r$, $B_r$. **B)** Each process $r \in R$ independently using the Basic sequential algorithm will construct the following point-to-set applications $Br^r_1(j_r)$ *for all* $j_r \in J_r$ *and* $Br^r_2(i_r)$ *for all* $i_r \in I_r$, the set of index pairs $GrBr^r_1 \cap GrBr^r_2$ and send these sets to the root process. If, as a result of the division of matrices $A$ and $B$, subgames $\Gamma_r = \langle I_r, J_r, A_r, B_r\rangle$ are generated, then $NE[\Gamma_r] = GrBr^r_1 \cap GrBr^r_2$; **C)** Root process using the sets $\{GrBr^r_1\}_{r\in R}$, $\{GrBr^r_2\}_{r\in R}$ and the REM functions, the graph $GrBr_1$ of the point-to-set application $Br_1$ and also the graph $GrBr_2$ of the point-to-set application $Br_2$ are built. The equilibrium profiles are all the profiles belonging to the intersection of the two given graphs $NE = GrBr_1 \cap GrBr_2$.

# Evolution of queuing system

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The theory of stochastic networks emerged from the need to exploit informational and numerical systems, economic networks and telecommunication networks. The processes, which describe the activity of these stochastic networks, can have a rather complex and large structure. The analysis and optimization of complex systems are multiparametric problems, being an important step in solving optimization problems, for complex systems under conditions of uncertainty and the risk of the primary mathematical model are usually stochastic. This, for example, appears in the theory of information processing networks.

A queuing system is the system in which the elements end up being processed. The terms "element" and "processing" are generic terms. An "element" could refer to customers. The elements could be customers arriving at a system, such as a bank, to receive services (to be "processed"). Another example is in the manufacturing process where an item might be a partially completed part that needs to be machined and so is sent to a station where it is properly machined when possible. Most of us pass through the streets of the city, driving cars or as passengers in a transport. The transport unit passing through an intersection is an element that needs to be served, and the service is provided by the intersection in the form of turning on the green light to go through it. In general, in a queuing system there is a physical or virtual location, sometimes moving, where items end up being processed. If the processor is available the element can be processed immediately, otherwise it must wait, if the processor is busy. The waiting element, which can occur if the processor is busy, seems to dominate the system descriptor, that is "string" or "waiting". In reality, this system can also be called a serving system.

Types of waiting systems:

1. waiting systems with a single node
2. waiting systems with multiple servers
3. waiting systems with multiple priorities
4. Polling waiting systems
5. tandem waiting systems
6. waiting systems $[SM|M|1|\infty]^N$
7. waiting systems $SM^{\varepsilon}|M|\infty$

The queuing system of the type $[SM|M|1|\infty]^N$ means that the input flow is described by the regeneration Markov process, the service time is exponentially distributed, n-servers are connected through a probability matrix. So, queuing networks are considered with semi-Markov flow. In the $SM^{\varepsilon}|M|\infty$ type waiting system the arrival times are the jump times of the semi-Markov process on a standard phase space $(E, \varepsilon)$ and the service time is exponentially distributed with a server, while the end of the servings are jumps of the Markov process when the waiting system is busy.

Studying the asymptotic averaging and diffusion approximation schemes for semi-Markov queuing systems given a random evolution, one uses an important feature for semi-Markov networks,

namely, the compensation operator of the corresponding extended Markov process. The results have a theoretical significance and a practical application in different fields of natural and social sciences and can be used in the study, analysis of various real service systems and networks, where waiting phenomena occur. The practical value of the results determines the possibility of their use in various fields of activity, namely, such as energy, economy, communications, medicine, social services, transport.

## A statistical analysis of the cybernetic feedback formula

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In a linear amplifier of the type $y = Ax$, made with many active components and with large dispersions, if a feedback is connected, with the transfer factor $\beta$ made with passive components and with small dispersions, the transfer factor on the entire loop is $\beta A$ and if the signal propagates practically instantaneously, the reaction system has as a transfer function the well-known cybernetic expression $T = A/(1 - \beta A)$. This is analyzed statistically, considering in turn the quantities $A$ and $\beta$ respectively as random variables described by probability density functions, relative to the Monte Carlo method, determining the probability density for $T$. Specific relations are used for the deterministic functional transformation of random variables. The case of the signal propagating with a certain delay is also discussed.

## Optimal maintenance policy

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Let $X(t)$ denote the state of a machine at time $t$. We assume that the process $\{X(t), t \geq t_0\}$ is a continuous-time Markov chain having the following states:

0 : the machine is working,
1 : the machine is undergoing routine maintenance,
2 : the machine is undergoing prolonged maintenance,
3 : the machine is being repaired.

The maintenance is carried out at random times. The time elapsed between two maintenance operations is an exponential random variable with parameter $u > 0$. We define

$$\tau(i) = \inf\{t > t_0 : X(t) = 3 \mid X(t_0) = i\}$$

for $i = 0, 1, 2$. We look for the value of $u$ that minimizes, under certain assumptions, the expected value of the cost criterion

$$J(i) = \int_{t_0}^{\tau(i)} (f(u) - \lambda)\, \mathrm{d}t,$$

where $f(u) > 0$ and $\lambda > 0$.

## Conditions for the preservation of Motzkin decomposability and the Pareto bordered property under addition

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We provide some sufficient conditions on pairs of Motzkin decompasable sets and Pareto bordered sets in order to get the Minkowski sum of their components Motzkin decomposable and Pareto bordered respectively.

We also prove that minimal faces of a closed convex set are also extreme faces of the set and vice-versa. This result allows us to define the generalized Minkowski sets by using the extreme faces. A Minkowski type theorem is proved with extreme faces playing the role of the extreme points in the classical Minkowski Theorem. The special class of Pareto bordered sets, which is a subclass of that of generalized Minkowski sets, is also taken into account. Indeed, as mentioned above, we show that the Minkowski sum of some Pareto bordered sets with the same lineality remains Pareto bordered. Note that the class of generalized Minkowski sets is not closed with respect to the Minkowski sum. It is however worth to mention that the class of closed convex sets which are both Motzkin decomposable and generalized Minkowski (or shortly, $MdgM$ sets) is closed both with respect to Minkowski sum and Cartezian product [J.E. Martínez-Legaz, C. Pintea, Closed convex sets that are both Motzkin and generalized Minkowski sets, J. Nonlinear Var. Anal. 8 (2024), No. 4, pp. 571-579].

## The fuzzy method for solving the multicriteria lineal optimization problem in integers

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In the proposed paper we focused on solving the multicriteria optimization model in integers [2], in which the objective functions are of lineal type. The mathematical model of the problem is the following:

$$\begin{cases} \left\{ \begin{array}{c} \min \\ \max \end{array} \right\} F_k(x) = \sum_{j=1}^{n} c_{kj} x_j, \quad k = \overline{1,r} \\ A \cdot x \leq b \\ x \in Z^+ \end{cases} \tag{1}$$

where: $A = \|a_{ij}\|$ is an array of size $m \times n (m < n)$, $C = \|c_{kj}\|$, is an array of size $r \times n (r < n)$, $x$ is a vector n-dimensional column, and $b$ is a $m$-dimensional column vector.

In order to solve the model (1) we will apply the fuzzy techniques [1]:

$$\mu_k(Z^k)) = \begin{cases} 1, \text{if } Z^k(x) \leq L_k \\ \dfrac{U_k - Z^k(x)}{U_k - L_k}, \text{ if, } L_k < Z^k(x) < U_k; \\ 0, \text{ if} Z^k(x) \geq L_k \end{cases} \tag{2}$$

where: $U_k$ is the worst upper bound and $L_k$ is the best lower bound of the objective function k, respectively.

Using the fuzzy techniques [1] we construct the set of problem, corresponding to (1) to solvie it iteratively and interactively depending on the aspirations of the decision maker. They can be described as follow:

Max $\lambda$ *in the same availability conditions and additionally:*

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij} + \lambda \cdot (U_k - L_k) \leq U_k, \quad k = \overline{1,r}, \quad x \in Z^{+} \tag{3}$$

The integer optimal solution of the model (3) is one of the optimal compromise solutions for model (1) that corresponds to the decision marker's aspirations.

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## A case when common sense fails to be true

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We study the following problem. Let $(X_n)_{n\geq 1}$ be a sequence of i.i.d. random variables, uniformly distributed on $[0,1]$ and $A \in \mathcal{B}([0,1])$ be a measurable set. We define $Z_n = (X_n, 1_A(X_n))$ with $1_A : \mathbb{R} \to \{0,1\}$, $1_A(x) = \begin{cases} 1 & if \quad x \in A \\ 0 & if \quad x \notin A \end{cases}$.

We say that the set $\mathcal{S}_n = \{Z_1, Z_2, ..., Z_n\}$ has a leader (respectively that it has an anti-leader) if there is a variable $Z_j \in \mathcal{S}_n$ such as $Z_j \geq Z_i$, $\forall 1 \leq i \leq n$ ( respectively if $\exists Z_j \in \mathcal{S}_n$ such as $Z_j \leq Z_i$, $\forall 1 \leq i \leq n$). Let $a_n$ be the probability that $\mathcal{S}_n$ has a leader, let $b_n$ be the probability that $\mathcal{S}_n$ has an anti-leader and $a = \lim\limits_{n\to\infty} a_n$, $b = \lim\limits_{n\to\infty} b_n$. If there are $Z_i, Z_j \in \mathcal{S}_n$ such as $Z_i \leq Z_k \leq Z_j$, $\forall 1 \leq k \leq n$ we say that $\mathcal{S}_n$ is ordered and we denote by $c_n$ the probability that it has this property. Let $c = \lim\limits_{n\to\infty} c_n$.

We are interested in the following questions:

a) What properties should the set $A$ have such that the limits $a, b, c$ do exist?

b) Is it true that $c = ab$?

# A New Approach for Escaping from Local Optimum Points in Genetic Algorithm Applied in the Traveling Salesman Problem

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Meta-heuristic algorithms are algorithms that have more advantages than exact and heuristic algorithms, so nowadays they have been received much attention by researchers and scientists for solving combinatorial optimizations problems. Because of using randomization concept in search for finding better solutions, this group has more effectiveness for escaping from local optimum and can get more quality solutions. The Genetic Algorithm is one of the most traditional algorithms used in most of the operation research problems because of its simplicity. In this paper, two novel approaches for this algorithm are proposed in which a first approach is a new cross and a second approach is to devote defined percentage of chromosomes in each iteration to haphazard solutions that leads the algorithm will have more ability for escaping from local optimum points. Conclusions on several standard instances of Traveling Salesman Problem show performance of this novel algorithm in compare to common Genetic Algorithm and other approaches.

**Keywords**: Genetic Algorithm, Traveling Salesman Problem, Premature Convergence, NP-Complete Problems

# 6. Algebra, Logic, Geometry (with applications)

# On Weyl tensor of six-dimensional planar Hermitian submanifolds of Cayley algebra

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It is known that every Gray–Brown 3-fold vector cross product in Cayley algebra induces an almost Hermitian structure on an arbitrary oriented six-dimensional submanifold [1]. Such almost Hermitian structures (in particular, Kählerian, nearly Kählerian, quasi Kählerian, Hermitian and special Hermitian) on six-dimensional submanifolds of Cayley algebra were studied by A. Gray [2], V.F. Kirichenko [3] and many others.

In this communication, six-dimensional planar Hermitian submanifolds of Cayley algebra are considered. The components of the Ricci tensor $ric$ and of the Weyl tensor $W$ (known also as the tensor of conformal curvature) for a six-dimensional Hermitian planar submanifold of Cayley algebra are computed.

We obtain the following results:

$$ric_{ab} = 0,\ ric_{\hat{a}b} = -2\mu^2\, T^7_{\hat{a}\hat{c}} T^7_{cb};$$

$$W_{abcd} = 0,\ W_{\hat{a}bcd} = 0,$$

$$W_{\hat{a}\hat{b}cd} = -\frac{\mu^2}{2}\left(T^7_{\hat{a}\hat{h}}\, T^7_{hc}\, \delta^b_d + T^7_{\hat{b}\hat{h}}\, T^7_{hd}\, \delta^a_c - T^7_{\hat{a}\hat{h}}\, T^7_{hd}\, \delta^b_c - T^7_{\hat{b}\hat{h}}\, T^7_{hc}\, \delta^a_d\right) + \frac{K}{20}\,\delta^{ba}_{cd},$$

$$W_{\hat{a}b\hat{c}d} = -\,2\,\mu^2\, T^7_{\hat{a}\hat{c}}\, T^7_{bd} + \frac{\mu^2}{2}\left(T^7_{\hat{a}\hat{h}}\, T^7_{hd}\, \delta^c_b + T^7_{\hat{c}\hat{h}}\, T^7_{hb}\, \delta^a_d\right) + \frac{K}{20}\,\delta^c_b\,\delta^a_d.$$

Here $\{T^7_{kj}\}$ are the component of the configuration tensor of the immersion of the six-dimensional planar Hermitian submanifold into Cayley algebra [4], $K$ is the scalar curvature of this submanifold; $a,\ b,\ c,\ d,\ h = 1,\ 2,\ 3$; $\hat{a} = a + 3$; $k,\ j = 1,\ 2,\ 3,\ 4,\ 5,\ 6$.

Knowing the components of the Weyl tensor for a six-dimensional planar Hermitian submanifold of the octave algebra, it is possible to apply this data to study so-called conformal analogs [5] of Gray¡92¿s identities [6] for this submanifold.

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## An application of some Kirichenko structural tensors of almost contact metric structure

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**1.** As it is known, an almost contact metric structure on an odd-dimension manifold $N^{2n+1}$ is defined by a system of tensor fields $\{\Phi,\ \xi,\ \eta,\ g\}$ on this manifold. Here $\xi$ is a vector, $\eta$ is a covector, $\Phi$ is a tensor of the type $(1,\ 1)$ and $g = \langle\cdot,\ \cdot\rangle$ is the Riemannian metric. Moreover, the following conditions are fulfilled:

$$\eta(\xi) = 1,\ \Phi(\xi) = 0,\ \eta \circ \Phi = 0,\ \Phi^2 = -id + \xi \otimes \eta,$$

$$\langle \Phi X, \Phi Y\rangle = \langle X, Y\rangle - \eta\left(X\right)\eta\left(Y\right),\ X, Y \in \aleph(N^{2n+1}),$$

where $\aleph(N^{2n+1})$ is the module of $C^\infty$-smooth vector fields on the manifold $N^{2n+1}$ [1]. Let us consider the so-called Cartan structural equations of an almost contact metric structure [2]:

$$d\omega^a = \omega^a_b \wedge \omega^b + B^{ab}_c\omega^c \wedge \omega_b + B^{abc}\omega_b \wedge \omega_c + B^a_b\omega \wedge \omega^b + B^{ab}\omega \wedge \omega_b,$$

$$d\omega_a = -\omega^b_a \wedge \omega_b + B^c_{ab}\omega_c \wedge \omega^b + B_{abc}\omega^b \wedge \omega^c + B^b_a\omega \wedge \omega_b + B_{ab}\omega \wedge \omega^b,$$

$$d\omega = C_{bc}\omega^b \wedge \omega^c + C^{bc}\omega_b \wedge \omega_c + C^b_c\omega^c \wedge \omega_b + C_b\omega \wedge \omega^b + C^b\omega \wedge \omega_b.$$

Here and further $\{\omega^\alpha\}$ and $\{\omega_\alpha\}$ are the components of the displacement forms ( $\omega_\alpha = \omega^{\hat{a}}$, $\omega^0 = \omega$); $\{\omega^a_b\}$ are the components of the Riemannian connection; $k,\ j = 1,\ \ldots,\ 2n$; $a,\ b,\ c = 1,\ \ldots,\ n$; $\hat{a} = a + n$;

$$B^{ab}_c = -\frac{i}{2}\Phi^a_{\hat{b},c};\ B^{abc} = \frac{i}{2}\Phi^a_{[\hat{b},\hat{c}]};\ B^a_b = i\ \Phi^a_{0,b};$$

$$B^c_{ab} = \frac{i}{2}\Phi^{\hat{a}}_{b,\hat{c}};\ B_{abc} = -\frac{i}{2}\Phi^{\hat{a}}_{[b,c]};\ B^b_a = -i\ \Phi^{\hat{a}}_{0,\hat{b}};$$

$$B^{ab} = i\ \left(\Phi^a_{0,\hat{b}} - \frac{1}{2}\Phi^a_{\hat{b},0}\right);\ B_{ab} = -i\ \left(\Phi^{\hat{a}}_{0,b} - \frac{1}{2}\Phi^{\hat{a}}_{b,0}\right);$$

$$C^a_b = -i\ \left(\Phi^0_{\hat{a},b} + \Phi^0_{b,\hat{a}}\right);\ C^{ab} = i\ \Phi^0_{[\hat{a},\hat{b}]};\ C_{ab} = -i\ \Phi^0_{[a,b]};$$

$$C^a = -i\ \Phi^0_{\hat{a},0};\ C_a = i\ \Phi^0_{a,0}.$$

Let us introduce the notation:

$$C^{abc} = \frac{i}{2}\Phi^a_{\hat{b},\hat{c}};\ C_{abc} = -\frac{i}{2}\Phi^{\hat{a}}_{b,c};\ F^{ab} = i\ \Phi^0_{\hat{a},\hat{b}};\ F_{ab} = -i\ \Phi^0_{a,b}.$$

The systems of functions
1) $F = \{F^k_j\}$, where $F^a_{\hat{b}} = F^{ab}$, $F^{\hat{a}}_b = F_{ab}$, and all other components of $F$ are zero;
2) $G = \{G^j\}$, where $G^a = C^a$, $G^{\hat{a}} = C_a$, $G^0 = 0$
determine tensors on the manifold $N^{2n+1}$ . In V.F. Kirichenko¡92¿ terminology [1], $F$ and $G$ are the fifth and the sixth structural tensors of an almost contact metric structure, respectively.

Structural form $\eta$ is closed if and only if the following conditions are fulfilled:

$$\Phi^0_{[a,b]} = \Phi^0_{[\hat{a},\hat{b}]} = 0,\ \Phi^0_{\hat{a},b} = \Phi^0_{a,\hat{b}} = 0,\ \Phi^0_{a,0} = \Phi^0_{\hat{a},0} = 0.$$

That is why we get

$$C^{ab} = 0,\ C_{ab} = 0,\ C^a_b = 0,\ C^a = 0,\ C_a = 0,$$

and

$$F^{ab} = 0, F_{ab} = 0.$$

So, we can formulate our main result.

**Theorem.** *The structural contact form of an almost contact metric structure is closed if and only if the fifth and the sixth Kirichenko structural tensors of this structure vanish.*

We remark that almost contact metric structures with closed structural forms are usually studied under the name of Kirichenko–Uskorev structures (see [3], [4] and others).

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## Semireflexive, left and right products of two subcategories

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We propose an synthesis of the authors' results concerning the semireflexiv product, the left product and the right product of two subcategories. The possibility of describing the R–semireflexive subcategories using the indicated products is shown.

**Keywords**: reflexives subcategories, coreflexives subcategories, semireflexives subcategories, semireflexives products, left and right products.

## Unitary units of modular group algebras

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The group algebra is a classical object in mathematics. Let $FG$ be the group algebra of a group $G$ over the field $F$. The set $V_*(FG)$ of unitary units (under the classical involution $*$) of the group of normalized units of the algebra $FG$ form a group which is called the unitary subgroup of the group algebra $FG$. In my talk we provide some recent results about the structure of the unitary subgroup $V_*(FG)$ such as nilpotency, locally nilpotency and so on (see [1, 2, 3, 6, 7, 8, 9]). We are discussing the connections of the structure of $V_*(FG)$ with other parts of mathematics (for example, see [4, 5, 10, 11]).

**Keywords**: group algebra; unitary unit; Novikov's problem.
**AMS 2010**: 16U60, 20D15, 20C05.

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# On topological $AD$-quasigroups

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The results of this study are related to works [2-5]. A groupoid $(G,\cdot)$ is called *medial* if it satisfies the law $xy\cdot zt = xz\cdot yt$ for all $x,y,z,t\in G$. A groupoid $(G,\cdot)$ is called *paramedial* if it satisfies the law $xy\cdot zt = ty\cdot zx$ for all $x,y,z,t\in G$.

A groupoid $(G,\cdot)$ is called *bicommutative* if it satisfies the law $xy\cdot zt = tz\cdot yx$ for all $x,y,z,t\in G$. A groupoid $(G,\cdot)$ is said to be *subtractive*, if the following conditions holds: $b\cdot(b\cdot a) = a$ and $a\cdot bc = c\cdot ba$ for all $x,y,z,t\in G$. A groupoid $(G,\cdot)$ is called *AD-groupoid* if it satisfies the law $a\cdot bc = c\cdot ba$ for all $a,b,c\in G$ .

A groupoid $(G,\cdot)$ is called a *groupoid Abel-Grassmann* or *AG-groupoid* if it satisfies the left invertive law $(a\cdot b)\cdot c = (c\cdot b)\cdot a$ for all $a,b,c\in G$. We define a *Ward groupoid* as any groupoid $(G,\cdot)$ containing an element $e\in G$ such that $a^2 = a\cdot a = e$ and $(a\cdot b)\cdot c = a\cdot(c\cdot(e\cdot b))$, for all $a,b,c\in G$.

A groupoid $(G,\cdot)$ is called a *quasigroup* if for every $a,b\in G$ the equations $a\cdot x = b$ and $y\cdot a = b$ have unique solutions [1].

The concept of $(n,m)$-identities was introduced by M.M. Choban and L.L. Chiriac in [3].

**Theorem 1.** *If $(G,\cdot)$ is an AD-multiplicative topological groupoid, $e\in G$ and the following conditions hold:*

*1. $xe = x$ for every $x\in G$,*

*2. $x^2 = x\cdot x = e$ for every $x\in G$,*

*3. if $xa = ya$ then $x = y$ for all $x,y,a\in G$,*

*then $(G,\cdot)$ is a subtractive and $Ward$ topological quasigroup with a $(2,1)$-identity $e$.*

**Corolary.** *If $(G,\cdot)$ is a subtractive quasigroup or Ward quasigroup, $e\in G$ and $xe = x$ for every $x\in G$, then $e$ is a $(2,1)$-identity.*

**Theorem 2.** *Let $(G,\cdot)$ be a topological AD-groupoid and $a*b = a\cdot(pb)$, where $p$ is fixed. Then $(G,*)$ is a commutative, bicomutative, medial and paramedial topological groupoid.*

**Theorem 3.** *Let $(G,+,\tau)$ be a commutative topological group. For $(x_1,y_1)$ and $(x_2,y_2)$ in $G\times G$ define*

$$(x_1,y_1)\circ(x_2,y_2) = (-x_1-y_1+y_2, -x_2-y_2+x_1).$$

*Then $(G\times G,\circ)$, is a non-associative, medial, AG and AD-topological quasigroup.*

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## Spectral spaces

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We use the terminology from [2].

The family $\mathcal{L}$ on subsets of the $Z$ is called a ring of sets, if $Z, \emptyset \in \mathcal{L}$ and for every $A, B \in Z$ we have that $A \cup B \in \mathcal{L}$ and $A \cap B \in \mathcal{L}$. The family $\mathcal{L}$ is a closed (open) base-ring of the space $X$, if $\mathcal{L}$ is a ring on sets and forms a closed (open) basis of the space $X$.

The space $X$ is called $pc$-space if for any closed irreducible set $F \subset X$ there exists a point $x \in X$ such that $F = cl_X\{x\}$.

For every topology $T$ on a set $X$ denote by $hT$ the topology generated by the subbase $T \cup \{X \setminus U : U$ is an open compact subset of $(X, T)$. Let $hX$ be the space $X$ endowed with the topology $hT$. The topology $hT$ is called the Hochster modification of the topology $T$.

The set $F$ is $h$-closed in $X$, if $F$ is closed in $hX$.

Let $Y \subset X$. The set $Y$ is $h$-dense in $X$ with respect to the $h$-modification of the topology of $X$.

**Definition 1.** *The space $X$ is called spectral space if:*

*1) $X$ is a compact $T_0$-space;*

*2) $X$ is a pc-space;*

*3) in $X$ there exists an open base ring of compact sets [3].*

**Theorem 1.** *For any space $X$ the following affirmation are equivalent:*

*1) $X$ is a spectral space;*

*2) $hX$ is a zero-dimensional compact space and in $X$ there is an open prebasis of compact sets;*

*3) $hX$ is a compact space and in $X$ there is an open prebasis of compact sets; 4) The space $X$ satisfies the conditions:*

*4.1) $X$ is a compact space;*

*4.2) the open and compact sets of space $X$ form an open basis of space $X$;*

*4.3) if $F$ is a closed set in $X$, $\xi$ is a family of open and compact sets and $\gamma = \{U \cap F : U \in \xi$ is a centered family, then $\cap\gamma = \emptyset$.*

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## On hyperbolic manifolds with fundamental groups generated by parabolic or hyperbolic translations

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A method of construction of the fundamental polytope for a complete non-compact hyperbolic manifold of dimension 5 of finite volume is proposed. Geometry of this manifold and its total geodesical submanifolds of different codimenions are investigated. Similar constructions of fundamental polytopes for manifolds with fundamental groups generated by translations (parabolic or hyperbolic) are discussed in different dimensions.

## The projected regions over the faces of a polyhedron

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We first characterize the region of the $n$-dimensional Euclidean space which is projected over a face of a closed convex subset of the Euclidean space in the same way it is projected over the affine hull of the face itself. The metric projection over such a face is the one associated to the entire closed convex set and the metric projection over the affine hull of such a face is the one associated to the affine hull. It turns out that this region is the closure of the inverse image, through the metric projection over the entire closed convex set, of the relative interior of the face.

We also characterize analytically the closure of the regions of $\mathbb{R}^n$ which are projected over the relative interiors of the faces of a polyhedral set, through the metric projection of the polyhedral set itself. We show that these regions are polyhedral sets by explicitly characterizing them through systems of linear inequalities.

## On self-orthogonal finite $n$-ary quasigroups

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The famous Euler conjecture about non-existence of orthogonal latin squares of order $n \equiv 2 \pmod 4$ was solved negatively in the middle of the 20th century, for every $n \geq 10$ of such type. This process led to new concepts, types of orthogonality and new problems in this area. A system of $n$-ary operations $A_1, A_2, ..., A_n$, defined on a set $Q$, is called orthogonal if the set of equations $\{A_i(x_1, x_2, ..., x_n) = a_i\}_{i=\overline{1,n}}$ has a unique solution for every $a_1, a_2, ..., a_n \in Q$. A system of $s$ $n$-ary operations defined on a set $Q$, where $s \geq n$, is orthogonal if any $n$ its operations are orthogonal.

The orthogonality of $n$-ary operations (quasigroups, latin hypercubes) was considered by many researches from algebraic and combinatorial point of view [1].

The operation denoted by ${}^{\sigma}A$, where $\sigma \in S_{n+1}$ and $A$ is an $n$-ary quasigroup operation, defined by the equivalence ${}^{\sigma}A(x_{\sigma 1}, x_{\sigma 2}, ..., x_{\sigma n}) = x_{\sigma n+1} \Leftrightarrow A(x_1, x_2, ..., x_n) = x_{n+1}$, is called a parastrophe of $A$. If $\sigma(n+1) = n+1$, than ${}^{\sigma}A$ is called a principal parastrophe. An $n$-ary quasigroup is called self-orthogonal if there exist $n$ orthogonal its principal parastrophes. It is known that there exist self-orthogonal latin squares of any order $q \neq 1, 2, 3, 6$ [2].

We consider new constructions of self-orthogonal finite $n$-ary quasigroups and give some estimations of their spectrum (possible order). In particular, using self-orthogonal quasigroups of arity $n$ and, respectively $m$, on a finite set $Q$, we construct self-orthogonal $nm$-quasigroups on $Q$. As corollaries it follows that there exist: 1) $2^k$-ary self-orthogonal quasigroups of every order $q \neq 1, 2, 3, 6$, where $k \geq 1$; 2) self-orthogonal $p^k$-quasigroups of prime order $p$, for every $p \geq 3$ and every $k \geq 1$; 3) self-orthogonal $2n$-quasigroups of order $q$, for every $q > 3, q \neq 6$ and $n + 1 \not\equiv 0 \pmod{q}$.

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## On ternary quasigoups with exactly three distinct and orthogonal parastrophes

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An $n$-ary groupoid $(Q, A)$ is an $n$-ary quasigroup if each of the elements $x_1, x_2, ..., x_{n+1}$ in the equality $A(x_1, x_2, ..., x_n) = x_{n+1}$ is uniquely determined by the remaining $n$ of them ($n \geq 2$). The operation ${}^{\sigma}A$, defined by the equivalence $A(x_1, x_2, ..., x_n) = x_{n+1} \Leftrightarrow^{\sigma} A = (x_{\sigma 1}, x_{\sigma 2}, ..., x_{\sigma n}) = x_{\sigma(n+1)}$, where $\sigma$ is a substitution of degree $n+1$, is called a parastrophe of $(Q, A)$.

It is proved in [1] that the number of distinct parastrophes of a ternary quasigroup divides 24. Also in [1] M. E. McLeish studied the possible sets of distinct parastrophes of a ternary quasigroup, using 24 identities in this quasigroup $A = {}^{\sigma_i}A, i = \overline{1, 24}$, denoted respectively, by $L_i, i = \overline{1, 24}$, where $\sigma_1 = (34), \sigma_2 = (12), \sigma_3 = (12)(34), \sigma_4 = (23), \sigma_5 = (14), \sigma_6 = (14)(23), \sigma_7 = (13), \sigma_8 = (13)(24), \sigma_9 = (24), \sigma_{10} = (123), \sigma_{11} = (132), \sigma_{12} = (124), \sigma_{13} = (142), \sigma_{14} = (134), \sigma_{15} = (143), \sigma_{16} = (243), \sigma_{17} = (234), \sigma_{18} = (1432), \sigma_{19} = (1423), \sigma_{20} = (1234), \sigma_{21} = (1243), \sigma_{22} = (1342), \sigma_{23} = (1324), \sigma_{24} = \varepsilon$.

**Lemma.** [1] *A ternary quasigroup $(Q, A)$ has exactly three distinct parastrophes if and only if it satisfies exactly one of the sets of identities:* $\Sigma_3^1 = \{L_1, L_2, L_3, L_6, L_8, L_{19}, L_{23}, L_{24}\}$, $\Sigma_3^2 = \{L_3, L_4, L_5, L_6, L_8, L_{21}, L_{22}, L_{24}\}$, $\Sigma_3^3 = \{L_3, L_6, L_7, L_8, L_9, L_{18}, L_{20}, L_{24}\}$.

Remark that in [2] the authors obtained a characterization of possible sets of distinct parastrophes of a ternary quasigroup using the subgroups of $S_4$.

Ternary quasigroups with exactly three distinct parastrophes, which are orthogonal, are studied in the present work.

**Theorem.** *Let $(R, +, \cdot)$ be an unitary, associative, commutative ring and let $(R, A)$ be a ternary linear quasigroup over $R$. Then $(R, A)$ has exactly three distinct and orthogonal parastrophes if and only of the operation $A(x_1, x_2, x_3)$ is one of the following: $\alpha x_1 + \alpha x_2 + Ix_3 + a$, $Ix_1 + \alpha x_2 + \alpha x_3 + a$ or $\alpha x_1 + Ix_2 + \alpha x_3 + a$, where $Ix = -x$, $\alpha \neq I$, $\alpha^2 = \varepsilon$, $\alpha a = Ia$, $\alpha, \alpha + \varepsilon \in Aut(R, +)$, $a \in R$.*

**Corollary 1.** *Let $(\mathbb{Z}_n, A)$ be a ternary linear quasigroup over $\mathbb{Z}_n$. Then $(\mathbb{Z}_n, A)$ has exactly three distinct and orthogonal parastrophes if and only if the operation $A(x_1, x_2, x_3)$ is one of the following: $\overline{a}x_1 + \overline{a}x_2 + \overline{n-1}x_3 + \overline{c}$, $\overline{a}x_1 + \overline{n-1}x_2 + \overline{a}x_3 + \overline{c}$ or $\overline{n-1}x_1 + \overline{a}x_2 + \overline{a}x_2 + \overline{c}$, where $(a, n) = 1$, $(a, n-1) = 1$, $a^2 \equiv 1 \pmod{n}$, $\overline{a}, \overline{c} \in \mathbb{Z}_n$.*

**Corollary 2.** *There exist ternary finite quasigroups of order $q$ with exactly three distinct and orthogonal parastrophes, for every odd $q \geq 3$.*

**Corollary 3.** *A linear ternary quasigroup $(\mathbb{R}, A)$, over the field of real numbers $(\mathbb{R}, +, \cdot)$, is idempotent with exactly three distinct and orthogonal parastrophes if and only if the operation $A(x_1, x_2, x_3)$ is one of the following: $x_1 + x_2 - x_3$, $x_1 - x_2 + x_3$ or $-x_1 + x_2 + x_3$.*

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## A note on the diophantine exponential equations

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One important topic in number theory is the study of Diophantine equations, equations in which only integer solutions are permitted. The field of Diophantine equations is ancient, vast, and no general method exist to decide whether a given Diophantine equation has any solutions, or how many solutions. The famous general equation $a^x + b^y = z^2$ has many forms. The literature contains a very large number of articles on non-linear such individual equations involving particular primes and powers of all kinds.

In this paper, we study the equations: $3^x + 18^y = z^2$, $3^x + 54^y = z^2$, where $x$, $y$, $z$ are non-negative integer numbers. One could cite here many articles on the equation $a^x + b^y = z^2$. We provide here only a small number of related equations which include the prime 3 in particular, such as [1, 2, 3, 4, 5, 6].

**Theorem 1**. *The exponential Diophantine equation $3^x + 18^y = z^2$ has exactly four integer solutions $(x, y, z) \in \{(1, 0, 2), (6, 3, 81), (8, 5, 1377), (10, 4, 405)\}$.*

**Theorem 2**. *The exponential Diophantine equation $3^x + 54^y = z^2$ has exactly four integer solutions $(x, y, z) \in \{(1, 0, 2), (3, 1, 9), (8, 3, 405), (14, 4, 3645)\}$.*

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# 7. Computer Science

## Reconstruction and noise reduction with conventional and deep learning algorithms in medical image processing

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Noise reduction in CT image processing has long relied on conventional techniques paired with various reconstruction methods. Recently, statistical model based, and deep learning algorithms have emerged as a powerful approach, showing significant improvements in reducing noise in CT scans [1, 2]. This study compares the effectiveness of three CT noise reduction strategies: a deep learning algorithm, a traditional method, and a hybrid approach combining both. We show the mathematical background of the procedures and evaluate their performance using the Catphan 600 phantom.

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## On some adaptive time step methods for a non-local model from Materials Science

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We shall present two different adaptive time step methods for a non-local version of the Allen-Cahn equation. Computational experiments are made to illustrate the effectiveness of the proposed methods, where the main goal is the computational cost.

## An artificial neural network approach for models in Biochemistry

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We shall present an artificial neural network approach to approximate the solutions of a system of nonlinear differential equations that arise in enzyme kinetics. Our method is based on a mixture of feed-forward artificial neural networks and optimization techniques. We perform several numerical experiments to show the efficiency of our method. Our approximation result is compared with the numerical solution obtained via the classical Runge-Kutta method.

## Enhancing user experience by integrating voice recognition technology in AR educational applications

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This paper explores the integration of augmented reality (AR) [1] and voice command technology in educational settings, specifically focusing on the interactive control of 3D animal models. By combining AR with voice commands, students can engage in immersive learning experiences, allowing them to visualize and manipulate digital animals in real-time. This approach enhances understanding of complex biological concepts, increases student engagement, and offers accessible learning opportunities for all students.

In AR applications, voice recognition [2] allows users to control digital elements without relying solely on physical controllers or touch-based interfaces. This hands-free interaction enables users to seamlessly navigate virtual environments, manipulate 3D objects, and perform various tasks using only their voice, a very useful thing both for people with a preferential auditory learning style and for people with physical disabilities.

By simply speaking commands, students can rotate, scale, or animate 3D animals, exploring different perspectives and behaviors. For example, a student might say, "Make the wolf roar," and the AR system would animate the 3D wolf accordingly, providing an interactive and auditory learning experience. Imagine a biology class where students are learning about animal behavior. With the combination of AR and voice commands, students could bring a virtual safari to life in their classroom. They could command a 3D wolf to walk, observe how a sparrow flies, or explore the movements of a fish in water. This interactive approach allows students to explore the characteristics and habitats of various animals in a controlled environment, fostering a deeper understanding of the subject matter.

The integration of voice recognition in augmented reality is poised to transform the way we interact with digital content and services. By leveraging the power of hands-free interaction, voice-enabled AR is opening up new possibilities for productivity, accessibility, and innovative user experiences.

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## Applying convolutional neural networks and visual transformers to satellite image data

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This paper explores the use and practicality of Vision Transformers (ViT) [1]in the context of classifying satellite imagery, inspired by the wide-spread using of transformers in language processing tasks, and related works in computer vision. The approach involved training a ViT model on the EuroSAT dataset [2] and comparing its performance with ResNet50 [3]. As expected, the results were disappointing, largely due to the inherent architectural strengths and weaknesses of the ViT model, such as scalability. Consequently, combining the widely used convolutional neural networks (CNNs) with ViTs together was a natural step in making use of both architectures and achieve better performance in satellite image classification practices than using CNNs alone.

The dataset used in this experiment is EuroSAT, containing 10 image classes provided by the Sentinel-2 satellite mission. The data is collected from several countries across Europe to different degrees, dominated by western European countries with The UK, France and Germany combined occupying approximately 47% of the entire dataset. The classes are as follows: Forest, River, Highway, Annual Crop, Seas or Lakes, Herbaceous Vegetation, Industrial, Residential, Permanent Crop, and Pasture. This breadth of image classes provides ample opportunity to create specific applications for land cover and use, particularly in agricultural or wild environments with seven out of three classes being reserved for such land uses.

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## 8. Education

## Interdisciplinary links in teaching higher mathematics at a technical university

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The article examines issues related to the development and implementation of interdisciplinary links when teaching Higher Mathematics to future engineers at a technical university. The strengthening of the role of the interdisciplinary approach both in the learning process and in scientific research is noted. In accordance with the new educational standards, the need to develop methodological approaches to teaching, which are based on the implementation of interdisciplinary links, where mathematics is given a special role, is emphasized. It has been established that the connection of mathematics with natural sciences, general technical and special professional disciplines allows to ensure a more complete assimilation of knowledge, forms skills and abilities helping future engineers to solve tasks related to their professional activities.

Tasks that must be solved to implement interdisciplinary links are formulated. The conditions under which these tasks can be successfully solved are formulated. The existing connections between Higher Mathematics and natural and general technical disciplines, which are studied in the first and second years of a technical university, are studied. Such disciplines as Electrical Engineering, Strength of Materials, Theoretical Mechanics, Physics are considered. The educational topics of the specified disciplines are compared with their mathematical content. Connections are presented in the form of tables, which reflect the correspondence between the nodal topic being studied and its mathematical support.

It is noted that the strengthening of interdisciplinary links in the study of Higher Mathematics and general technical and special professional disciplines at the technical university is one of the important and necessary conditions for the formation of professional competencies of future engineers. The proposed variant of implementation of interdisciplinary connections forms students' competences for solving professionally oriented problems in the future, and thus prepares the future specialist-engineer for the challenges of modern production.

**Keywords:** interdisciplinary links, Higher Mathematics, technical university, engineering education.

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## New challenges in mathematics and computer science education: case study - Republic of Moldova

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Mathematics and computer science education in the Republic of Moldova is undergoing continuous transformations, driven by technological advancements as well as global and local developmental trends and social challenges: the increasing frequency of crises, the emergence of the need

to research trans- and inter-disciplinary topics, the declining interest of the younger generation in the natural and real sciences, the shortage of teaching staff, decreasing learning motivation, modest ratings in international assessments such as PISA, etc.

The current state of this education has been analyzed from the perspective of curriculum changes, methodological support for teacher training, and new challenges generated by the mentioned transformations. It has been found that the seven centers for continuous professional development, which target mathematics and computer science teachers in general education, offer 22 training programs that cover only about 50% of the new learning topics stipulated in the curriculum through elective courses. At the same time, alongside potential perspectives for individualized learning, adapting educational content to the national and international socio-cultural context, there is a likelihood of various risks and challenges arising: resistance to change, inadequate teacher training, lack of resources and infrastructure, curriculum overload, challenging and inefficient assessment, differentiated adaptation to student needs, coordination and coherence of transformational processes, insignificant impact on student performance, etc.

To mitigate these risks, continuous support is proposed for mathematics and computer science teachers by reconceptualizing teacher professional development programs, which will include learning topics focused on new pedagogical models, tools, and methodologies aimed at increasing learning motivation and involving students in the educational process, such as Conditional knowledge strategy, realistic mathematics and computer science education, the flipped classroom method, video pedagogy, virtual reality technologies, augmented reality, mixed reality, etc.

**Keywords**: education, mathematics, informatics, curriculum.

## Interdisciplinarity – key element in forming programming skills

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Interdisciplinarity between mathematics and computer science enhances problem-solving capabilities by applying mathematical theories and models to computational problems and algorithms. Collaboration between mathematics and computer science fosters innovation, as mathematical methods are often used to optimize algorithms and improve software design, leading to more effective and robust technological solutions. This article examines the role of mathematical concepts from set theory in solving computer science problems. It presents the basic algorithms needed to understand the data type "set" and how these concepts can contribute to the efficiency and clarity of computational solutions.

**Keywords**: interdisciplinarity, set, set operations, structured data types, set type, algorithm analysis.

## Methodology of teaching systems of equations

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The study of mathematics has an important role in the formation and development of students' personality, in the acquisition of skills necessary for lifelong learning, but also for integration into

a knowledge-based society. It contributes to the formation and development of students' ability to formulate and solve problems, including those from everyday life, based on knowledge from different fields. Most problems involve using real numbers or solving them with equations and systems of equations. Systems of equations play an important role in our lives, so we must pay special attention to this important chapter of mathematics. Systems of equations are mathematical models of many physical, chemical, etc. phenomena, then solving various problems, including those we encounter in practice, ultimately comes down to solving systems of equations. Therefore, the objectives of this method consist in knowing the phenomena, processes of reality and obtaining a method for solving many practical and scientific problems.

The most important aspects that must be highlighted in the study of the given method are:

- the purposes of studying the given method
- the essence and content of the method
- the stages of method formation.

Analyzing the national curriculum for the discipline of Mathematics, we will notice that students are hungry for knowledge and skills to solve equations from the primary grades. They do not use the notion of an equation, but solve exercises of the type: 5+ =2.

The most widespread methods of introducing the notion of equation and system of equations in secondary school is to start from a problem from everyday life and put it into an equation or system. After that, the notions of system of equations and solution of the system of equations are already explained. The types of systems of equations are defined: incompatible system, determined compatible system and indeterminate compatible system. Then we move on to methods of solving equations and systems of equations.

According to the school curriculum, students become familiar with systems of equations with 2 unknowns in the 8th grade. The methods of solving systems of equations studied in secondary school are:

I. Method of substitution

II. The reduction method

In the 11th grade, systems of n-equations with m-unknowns are studied. The methods for solving systems of equations in high school are:

I. Cramer's method (for the humanistic profile)

II. Gauss's method and the matrix method (for the real profile).

## Intervale de încredere - aplicaţii rezolvate în R

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Se prezintă expresia intervalului de încredere pentru medie şi forma generală a intervalului de încredere pentru dispersie în cazul când eşantionul se formează pe baza unei selectţi simple aleatoare formată prin extrageri independente. Pentru exemplele prezentate sunt oferite solutţile matematice, dar şi programele implementate în limbajul R.

## Applications of straight lines in three-dimensional Euclidian space solved in Maple

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A method for approaching some analytical geometry problems with the help of the Maple environment, related to some applications with straight lines in the three-dimensional Euclidean space, problems that classically solved involve the application of the same formula several times and a lot of time would be lost is presented. First, the classical solution method is presented (for some applications without complete solution), using the application of the formulas presented in the Analytical Geometry or Computational Geometry courses, then, the same problems are solved with the help of the Maple environment, presenting the graphic representation of the solution too.
**Keywords**: straight line, distance, Maple

## Literacy in relation to problem-solving competence

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In the article is analyzed the relationship between the formation and development of literacy competence in trainees and the development of problem-solving and programming competencies. Additionally is elucidated, the importance of the educator's ability to correctly formulate a problem and its impact on literacy and the process of developing problem-solving skills in trainees. The positive effects of literacy and its impact on cognition are highlighted.
**Keywords**: literacy, problem-solving competence, lifelong learning, programming, academic achievement, literacy strategies, functional illiteracy.

## Statistical analysis of nominal or ordinal variables in pedagogical research

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In the paper, the implementation of the $\chi^2$ test of association for nominal variables is exemplified and the methodology of processing variables with multiple answers is described, cases less common in pedagogical research in Moldova. The digital tools used are SPSS and Jamovi software.
**Keywords**: pedagogical research, ordinal variables, nominal variables, non-parametric tests, statistical data, SPSS, Jamovi.

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## Specificity of implementation of bilingual education in Ukrainian universities

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The article highlights issues related to the implementation of bilingual education in Ukrainian higher education institutions. The study of this problem promotes to the integration of Ukraine into the European and world cultural and educational space, which is one of the priority areas of education development according to the National Doctrine of Education Development.

Specific features acquired by bilingual education at the end of the 20th century are outlined. Bilingual education is both a means of obtaining bilingual education and a process of forming a personality open to interaction with the surrounding world. It is noted that during bilingual education, a foreign (in particular, English) language is not only the goal, but also a means of understanding the world of special knowledge.

The peculiarities of the implementation of bilingual education in higher education institutions are considered in detail. There have been formulated the questions which need to be theoretically comprehended for the successful implementation of bilingual education in higher educational institutions. Special attention is paid to the set of didactically and methodologically relevant factors, due to the peculiarities of the taught discipline and the future specialty of the student. Appropriate criteria (both linguistic and non-linguistic) have been identified, which affect the content, organization and choice of methods of bilingual education in a certain specialty. The need to identify and take into account all relevant factors in their interrelationships is established, because only in this case is it possible to create a scientifically based concept of bilingual education at one or another non-language faculty in a higher education institution.

**Keywords**: foreign language education, bilingual education, bilingual professional education, model of bilingual education, subject-oriented didactic models, higher mathematics, academic mobility.

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## Teaching robotics through virtual laboratories

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In the context of modern STEM education, using virtual labs to teach robotics has become an effective and affordable way to introduce students to fundamental engineering and programming concepts. This article examines various virtual lab platforms, including TinkerCAD Circuits, VEXcode VR, Robot Virtual Worlds, RoboBlockly, and Webots, with a particular focus on OpenRobertaLab - an open-source platform developed by Fraunhofer IAIS, notable for its intuitive interface and ability to support a wide range of educational robots, such as LEGO Mindstorms and micro. The article explores the advantages and challenges of using these platforms in the educational process, highlighting how OpenRobertaLab facilitates interactive and accessible robotics learning.

**Keywords**: robotics, virtual laboratory, simulators, OpenRobertaLab.

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## Aspects of integrating VR and AR technologies in steam education

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This article examines the impact of integrating virtual and augmented reality technology in the field of STEAM education. It highlights current trends and benefits of using virtual and augmented reality, such as immersive experiences, interactive visualization, hands-on, personalized, and collaborative learning. Recent studies demonstrate the applicability of these technologies in STEAM learning and how virtual and augmented reality are revolutionizing education. The article explains the advantages and limitations of integrating virtual and augmented reality into STEAM education, along with the necessary conditions for achieving effective STEAM learning.

Additionally, it emphasizes the relevance of fundamental principles for adapting to the virtual environment and suggests ways to overcome STEAM learning challenges through partnerships, ongoing teacher training, and the creation of online communities for sharing best practices.

**Keywords**: virtual reality, augmented reality, STEAM education, innovative technologies.

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