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CAIM 2026, Chişinău, August 20–23, 2026

Fixed Point Theory in Machine Learning: the role of nonexpansive type mappings

Vasile Berinde

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Fixed point iterative algorithms are important in statistics and data science, mainly due to their simple and natural design, specific framework for ensuring weak / strong convergence, as well as applicability to high-dimensional problems.
Most of the problems in this area can be modeled as fixed point problems that involve nonexpansive type mappings.
Our aim in this paper is to give an overview on some recent fixed point type algorithms with acceleration techniques for variational inequalities and demicontractive mappings with illustrative applications in cybersecurity investments and classification problems.

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Factorization structures. History and Results

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This paper presents a historical and mathematical overview of the development of factorization structures in category theory, viewed through the author's research journey beginning in 1967

under Professor D. A. Raicov in Moscow. Starting from the connection morphism problem in additive categories and Raicov's notion of semiabelian category, the paper traces the introduction of orthogonal morphisms and the general theorem:

A morphism α is considered orthogonal from above to a morphism β and β orthogonal from below to it α if for any commutative square $\gamma \cdot \alpha = \beta \cdot \delta$ there exists its diagonal φ such that: $\delta = \varphi \cdot \alpha$, $\gamma = \beta \cdot \varphi$

For any class of morphisms $\mathcal{A}$, the notations were introduced: $\mathcal{A}^{\top}$ - all orthogonal morphisms from above to any morphism in $\mathcal{A}$; $\mathcal{A}^{\perp}$ - all orthogonal morphisms from below to any morphism in $\mathcal{A}$; $\mathcal{A}^{\urcorner} = \mathcal{E}pi \cap \mathcal{A}^{\top}$, $\mathcal{A}^{\llcorner} = \mathcal{M}ono \cap \mathcal{A}^{\perp}$. The author observed that for any factorization structure $(\mathcal{P}, \mathcal{I})$, $\mathcal{P} = \mathcal{I}^{\urcorner}$ and $\mathcal{I} = \mathcal{P}^{\llcorner}$. For right factorization structures $(\mathcal{P}, \mathcal{I})$, $\mathcal{P} = \mathcal{I}^{\urcorner}$ and $\mathcal{I} = \mathcal{P}^{\perp}$. For left factorization structures $(\mathcal{P}, \mathcal{I})$, $\mathcal{P} = \mathcal{I}^{\top}$ and $\mathcal{I} = \mathcal{P}^{\llcorner}$.

Theorem 1. *In a small locally and colocallly category with projective (respectively inductive) limits for any class of epi $\mathcal{A}$ (respectively for any class of mono $\mathcal{A}$) the pair $(\mathcal{A}^{\perp\urcorner}, \mathcal{A}^{\perp})$, respectively $(\mathcal{A}^{\urcorner}, \mathcal{A}^{\urcorner\perp})$, is a left (respectively right) factorization structure. In particular, in a small locally and colocal category with projective or inductive limits for any class of epi $\mathcal{E}$ and any class of mono $\mathcal{M}$ the pairs $(\mathcal{E}^{\perp\urcorner}, \mathcal{E}^{\llcorner})$, and $(\mathcal{M}^{\urcorner}, \mathcal{M}^{\urcorner\llcorner})$ form factorization structures.*

The main results concern the category $\mathcal{C}_2\mathcal{V}$ of Hausdorff locally convex topological vector spaces: the description of the factorization structure $(\mathcal{E}_p, \mathcal{M}_u)$ =(the class of exact epimorphisms, the class of universal monomorphisms), the existence of smallest and largest factorization structures associated with any nonzero reflective subcategory, and the theory of semireflexive and c-reflective subcategories, including the semireflective product and the operation $\rho(\mathcal{L}, \mathcal{R})$ - the full subcategory of all objects X for which $rX = lX$, where $\mathcal{L}$ and $\mathcal{R}$ are reflective subcategories. It is proved that if $\mathcal{L}$ is c-reflective, then $\rho(\mathcal{L}, \mathcal{R})$ is an $\mathcal{L}$-semireflexive subcategory.

The commutative semigroup $\mathbb{V}_{pb}$ of reflector functors is also described.

The paper concludes with an open problem on τ-fine and τ-closed morphisms in the category of Tychonoff spaces, asking for which cardinals τ these classes form a factorization structure.

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Monoid Algebras in Light of Skew Polynomial Rings

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Given a non-commutative associative ring A, a multiplicative monoid G, and a family σ of mappings $A \to A$ satisfying some suitable axioms (called a $D-$structure), a monoid algebra $A\langle G\rangle$ was introduced in [1], see also [2]. This construction naturally generalizes classical Ore extensions, encompasses results previously obtained by Smits [3], and extends to other related structures, see [4]. The obtained results have applications to the study of skew polynomial rings and others related algebraic structures.

Skew polynomial rings constitute a significant source of important applications. These are polynomial rings over a ring A with identity in which the indeterminate does not commute with the elements of A, but the multiplication is controlled by an endomorphism and/or a derivation (or a suitable generalization thereof) of A. The generalized monoid rings $A\langle G, \sigma\rangle$ associated with D-structures σ are analogous to skew polynomials rings with the multiplication controlled by a family of self-maps of A indexed by the elements of G. Furthermore, the generalized monoid ring introduced in [1] can be treated from more general aspect, see for instance, [5].

We introduce the concept of a (generalized) skew polynomial ring over a ring A, defined as a ring obtained by equipping the additive group of the polynomial ring over A with a new multiplication. One approach to constructing such rings is via a D-structure, which, in this setting, arises as a special case of the generalized monoid algebra concept/construction. This is a consequence of the more general result proved in [6] that every generalized monoid ring is definable by a D-structure.

Theorem 1. *Every skew polynomial ring can be defined by a D-structure.*

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The fear factor: predation risk and aggregation dynamics in a three-species food chain model

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We present a qualitative investigation of a three-species food chain model incorporating modified Cosner and Holling Type II functional responses, respectively, primarily from a stability and bifurcation viewpoint. The modified Cosner functional response captures group hunting by intermediate predators on a schooling prey population, together with the time lost by intermediate predators while evading top predators during foraging. Fear of top predators is further incorporated by reducing the top-intermediate predators encounter rate, while fear of intermediate predators is modeled as a reduction in prey growth rate.

Our qualitative analysis reveals several instances of bistability and tristability, along with the occurrence of certain distinct types of codimension-one bifurcations. Comprehensive numerical simulations, obtained by varying either one or two ecologically relevant parameters, reveal the complex dynamics of the system. Our results show that increasing the fear of intermediate predators in

the prey population to moderate levels promotes coexistence through period-halving bifurcations, whereas both the absence of fear and excessively high fear levels drive, respectively, the extinction of both predator species and of the top predator alone. We further find that moderate reductions in the top-intermediate predators encounter rate and in the time lost by intermediate predators due to fear of top predators favor coexistence.

Keywords *Food chain model; fear effect; bistability; tristability; bifurcations; modified Cosner functional response.*

MSC codes. 37N25, 92B05, 34C60, 34C25, 92D25.

Permanent function: a tool to solve combinatorial problems

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Two functions in matrix theory, determinant and permanent, look very similar:

$$\det A = \sum_{\sigma \in S_n} (-1)^{\sigma} a_{1\sigma(1)} \cdots a_{n\sigma(n)} \quad \text{and} \quad \operatorname{per} A = \sum_{\sigma \in S_n} a_{1\sigma(1)} \cdots a_{n\sigma(n)}$$

here $A = (a_{ij}) \in M_n(\mathbb{F})$ is an $n \times n$ matrix, and S_n is the set of all permutations of the set $\{1, \ldots, n\}$.

These functions are in the center of investigations for several centuries because of their numerous applications and a number of open mathematical problems. While the computation of the determinant can be done in a polynomial time, it is still an open question, if there are such algorithms to compute the permanent. Consequently, the question of the values that permanent can achieve, is actual. Also the notion of the permanental rank defined as the size of the maximal submatrix with a non-zero permanent is known. We will talk about recent results in these directions including the following three interrelated topics.

In 1974 Wang [4, Problem 2] posed a question to find a decent upper bound for $|\operatorname{per}(A)|$ if A is a square $(-1, 1)$-matrix of rank k, see also Problem 6 in [3, Chapter 8.4]. In 1985 Kräuter [2] conjectured a certain upper bound. We proved the Kräuter's conjecture and thus obtain the complete answer to the Wang's question as well as characterized the matrices with the maximal possible permanent for each value of k.

We will discuss an improvement of the Brualdi-Newman bound [1] for the consequent values of permanent of $(0, 1)$ matrices will be also presented.

Various estimates for the linear subspaces consisting of matrices of bounded permanental rank will be considered.

The talk is based on the joint works with my students M. Budrevich, D. Dryabin, I. Spiridonov, and K. Taranin.

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Coloring, Circuits, and Catastrophes: Extremal Bounds in Simple Line Arrangements

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Arrangements of n lines in general position are often introduced through the classical inductive count of regions in the plane. Yet beyond this elementary enumeration lies a richer combinatorial and topological structure, involving triangular faces, Hamiltonian cycles in the dual graph, and polygonal configurations induced by the arrangement. In this article, we revisit line arrangements from a dynamical perspective, emphasizing how their apparent rigidity can be understood through continuous deformation. The central tool is the Catastrophe Lemma, which provides a natural framework for tracking how the face structure changes under controlled perturbations of the lines. This approach yields a motivated proof of the sharp $n-2$ lower bound on the number of triangular regions, replacing a purely static argument with one based on the evolution of the arrangement. We then examine the dual graph of the arrangement and discuss the parity phenomena that obstruct the existence of Hamiltonian cycles, showing how global topological constraints emerge from local incidence data.

1. Partial Differential Equations

3D Medical Imaging Frameworks Combining Deep Neural Networks to Partial and Ordinary Differential Equations

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This research work surveys 3D medical image analysis techniques that combine effectively deep learning models to the differential equations. The partial differential equation (PDE) - based models are applied successfully in the 3D medical image preprocessing and segmentation fields.

The nonlinear vector-valued diffusion models used for 3D medical image filtering are described first. Our own contributions in this domain, representing second and fourth-order nonlinear vector-valued diffusion-based filters for Gaussian, Poisson and mixed noises, which could be applied successfully for MRI denoising and restoration, are described first.

Deep Learning models for medical imaging based on ordinary differential equations (ODE) are presented next. These hybrid DL-based models combine the learning capacity of the neural networks to robust and interpretable priors coming from classical mathematics. Modern deep learning architectures for 3D medical image analysis, which are based on Neural ODEs (Neural Ordinary Differential Equations) and Reaction-diffusion Networks, are surveyed here.

Our own contributions in the medical image segmentation domain are also described in this work. Thus, the nonlinear anisotropic diffusion-based models can be used successfully to improve the deep neural networks for MRI segmentation, such as the U-Net model. Some enhanced hybrid U-Net architectures for 3D medical image segmentation, which incorporate PDE-based layers and detect successfully the brain tumors, are presented here.

Keywords *3D medical image analysis, vector-valued diffusion-based filters, partial and ordinary differential equations, Neural ODEs, reaction-diffusion model, deep segmentation network, hybrid 3D U-Net architecture, PDE layer, tumor detection*

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On a class of semi-linear hyperbolic system

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This paper investigates the behavior of solutions for a class of one-dimensional semilinear hyperbolic systems with source terms. These systems arise in the modeling of soil erosion by water under uniform flow dynamics. Using the method of characteristics, we derive both exact analytical and numerical solutions. Furthermore, we establish explicit conditions on the model parameters that guarantee the positivity of the solutions and determine a precise upper bound required by the physical constraints of the system.
Acknowledgments. This work was carried out within the framework of the "Analysis and Control of Differential Systems" research program of the Romanian Academy.

A Fast Second-Order Fractional-Steps Approach for Cubic Nonlinear Reaction-Diffusion Problems

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We present a second-order fractional-steps method for a reactiondiffusion problem featuring a cubic nonlinear reaction term and nonhomogeneous Robin boundary conditions. A corresponding algorithm is formulated to approximate the solution of this nonlinear parabolic problem. Numerical experiments are conducted to demonstrate the effectiveness of the proposed approach, with a particular focus on computational efficiency. Finally, the performance is compared against the results obtained via the iterative Newton method.

Learning the Parameters of a Nonlocal Anisotropic Reaction–Diffusion Neurovascular Model from the BOLD Signal

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Functional Magnetic Resonance Imaging (fMRI) probes brain activity non-invasively through the Blood-Oxygen-Level-Dependent (BOLD) signal, which arises from the neurovascular coupling between neuronal firing, metabolic demand and the local hemodynamic response. Reaction–diffusion models of neuronal activity coupled to a hemodynamic "balloon" model reproduce the spatiotemporal BOLD response well [1,3,4], yet their predictive value hinges on physical parameters that are almost always fixed by hand. The forward problem is by now well understood; the open question we address is the *inverse* one: to what extent can these parameters be learned from the BOLD signal alone? Unlike the region-based Dynamic Causal Modelling framework [4], we retain the full spatial structure of a reaction–diffusion field, including anisotropic and nonlocal diffusion that region-based methods cannot natively represent.

We study a coupled neurovascular model in which a neuronal activity field $N(\mathbf{x}, t)$ on a cortical patch $\Omega \subset \mathbb{R}^2$ obeys an anisotropic reaction–diffusion equation in the sense of [2],

$$P_{sp}\,\partial_t N = P_d\big[K_u(|\nabla N|)\,\Delta N + \nabla K_u(|\nabla N|)\cdot\nabla N\big] + P_r\,R(N) - \alpha_N N + P_s\,I(\mathbf{x}, t), \qquad (1)$$

with the gradient-dependent (edge-stopping) mobility $K_u(\sigma) = \big(1 + (\sigma/\lambda)^2\big)^{-1}$, a known external drive $I(\mathbf{x}, t)$, and a reaction term $R(N)$ taken either as the logistic (monostable) form $3N(1-N)$ or the cubic bistable form $N - N^3$. The neuronal field is coupled, through a vasodilatory signal, to blood flow f, blood volume v and deoxyhemoglobin q via the balloon hemodynamics of [1,3], and the observable signal is the semi-empirical readout $\mathrm{BOLD} = V_0\big[k_1(1-q) - k_2(1-q/v) + k_3(1-v)\big]$. The local diffusion in (1) may optionally be replaced by a convolution-kernel (nonlocal) term $\int_\Omega J(\mathbf{x}-\mathbf{y})\,[N(t,\mathbf{y}) - N(t,\mathbf{x})]\,d\mathbf{y}$ representing long-range, white-matter–mediated connectivity.

We formulate parameter estimation as a PDE-constrained inverse problem, minimizing the misfit between simulated and observed BOLD subject to the coupled dynamics. The forward model is integrated with a *differentiable* fractional-steps numerical scheme endowed with adaptive time stepping, carried over from [2]; reverse-mode automatic differentiation through the unrolled scheme yields exact gradients (the discrete adjoint), which drive a gradient-based optimizer, benchmarked against a derivative-free baseline. Crucially, the estimation is restricted to the stability (Courant–Friedrichs–Lewy) and uniform-parabolicity regime of the scheme, so that every candidate model produced during learning is guaranteed to remain well posed.

A central obstacle is identifiability: only BOLD is observed, and only through the hemodynamic state, which is a smoothed, delayed transform of N. A preliminary scaling and redundancy analysis removes non-identifiable parameter groups – in particular the speed parameter P_{sp}, which is exactly redundant with the overall scale of the neuronal coefficients, while the input gain remains confounded with the neurovascular coupling and, near baseline, the reaction rate with the passive decay. This reduces the learnable set to $\hat{\theta} = \{P_d, P_r, P_s, \lambda, \tau_0, \alpha\}$. We then carry out an extensive synthetic-recovery study: BOLD data are generated from known parameters, corrupted by realistic thermal and physiological noise across a range of signal-to-noise ratios, and the parameters

are re-estimated. Combining recovery statistics with a Fisher-information and profile-likelihood analysis [5], and a Laplace posterior for uncertainty, we build an *identifiability map* that quantifies which parameters can be recovered reliably and which remain mutually confounded – the diffusion and mobility contrast being separable only when the activity field develops fronts, and the transit time and Grubb exponent forming a likelihood ridge.

The study delineates the practical limits of data-driven calibration for neurovascular reaction–diffusion models, provides a reproducible methodology bridging rigorous PDE analysis with parameter learning, and paves the way toward fitting real task-based and resting-state fMRI recordings, with applications to pathological dynamics such as seizure-front propagation in epilepsy and perfusion deficits in stroke.

Keywords*Reaction–diffusion model; neurovascular coupling; BOLD/fMRI; parameter identifiability; PDE-constrained optimization; differentiable solver; fractional-steps method; anisotropic and nonlocal diffusion.*

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The regular relaxation of the Cauchy-Dirichlet initial boundary problem for the Cahn-Hilliard equation

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Let $\Omega \subset \mathbb{R}^n\,(n=1,2,3)$ be a bounded domain with smooth boundary $\partial\Omega$, $T>0$ and $Q_T = \Omega \times (0,T)$. Our goal is to study the behaviour of the solutions of the initial boundary problem

$$\left\{\begin{array}{l} \varepsilon\,\partial_{tt}^2 u_\varepsilon + \partial_t u_\varepsilon - \Delta\big(-\Delta u_\varepsilon + a\,u_\varepsilon + \delta\,\partial_t u_\varepsilon + B(u_\varepsilon)\big) = f_\varepsilon(x,t), \quad (x,t)\in Q_T, \\ u_\varepsilon\big|_{t=0} = u_{0\varepsilon}(x), \quad \partial_t\,u_\varepsilon\big|_{t-0} = u_{1\varepsilon}(x), \quad x\in\Omega, \\ u_\varepsilon\big|_{x\in\partial\Omega} = \Delta\,u_\varepsilon\big|_{x\in\partial\Omega} = 0, \quad t\in(0,T), \end{array}\right. \tag{P_ε}$$

as $\varepsilon \to 0$, where $\Delta : H^2(\Omega) \bigcap H_0^1(\Omega) \mapsto L^2(\Omega)$, $a \in \mathbb{R}$, $\delta \geq 0$ and $B(u) = |u|^p u$. The problem (P_ε) is the relaxation for the corresponding initial boundary problem for the Cahn-Hilliard equation

$$\begin{cases} \partial_t v - \Delta\big(- \Delta v + a\, v + \delta\, \partial_t v + B(v)\big) = f(x,t), \quad (x,t) \in Q_T, \\ v\big|_{t=0} = v_0(x), \quad x \in \Omega, \\ v\big|_{x \in \partial\Omega} = \Delta\, v\big|_{x \in \partial\Omega} = 0, \quad t \in (0,T). \end{cases} \tag{P_0}$$

In the case $\delta = 0$ system (P_ε) is non viscous hyperbolic relaxation for the system (P_0), and, respectively, the viscous relaxation for the Cahn-Hilliard equation in the case $\delta > 0$.

The model (P_0) is governed by the celebrated Cahn-Hillard equation, which was proposed to describe phase separation phenomena in binary systems (phase separation process between healthy and tumour, the phase separation of a binary mixture and other phenomena) [J.W. Cahn and J.E. Hilliand, 1958]. The operator B is the derivative of a non-convex potential $\mathcal{B}$.

Under some smoothness conditions on the functions $u_{0\varepsilon}$, $u_{1\varepsilon}$, f_ε, v_0, f, if

$$||u_{0\,\varepsilon} - u_0||_{H_0^1(\Omega)} \to 0,\ ||u_{1\,\varepsilon} - u_1||_{L^2(\Omega)} \to 0,\ ||f_\varepsilon - f||_{W^{1,2}(0,T;L^2(\Omega))} \to 0,$$

using the relationship between the solutions of systems (P_ε) and (P_0), as well as a priori estimates of the solutions to the system (P_ε), we prove that

$$u_\varepsilon \to v \quad \text{in} \quad C([0,T]; L^2(\Omega)) \cap L^\infty(0,T; H_0^1(\Omega)), \quad \text{as} \quad \varepsilon \to 0.$$

This means that the perturbations of the system (P_0) by the system (P_ε) are regular in the indicated norms.

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2. ODEs; Dynamical Systems

Nonlinear Dynamics and Bifurcations in Nuclear-Driven Hydrogen Production Systems

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We study a three-component kinetic model describing a nuclear-hydrogen-production-type process, extended by a nonlinear feedback term of the form $\alpha y^2 z$. We analyze the equilibrium structure and linear stability using the Routh–Hurwitz criterion and prove, both analytically and numerically, that the baseline extended model (3) admits **NO Hopf bifurcations**: the Routh determinant $c_1c_2 - c_3$ stays strictly positive over the entire physical parameter domain, and the observed instabilities are of saddle-node type (multistability, hysteresis) rather than oscillatory. Even the introduction of separated time scales yields only divergent behaviour, not a stable limit cycle. We then show that replacing the polynomial feedback by a saturable (Michaelis–Menten-type) kinetics,

$$\frac{\alpha y^2}{1+\kappa y^2},$$

qualitatively changes the dynamics: a genuine Hopf bifurcation appears. A full normal-form reduction (projection method) yields a POSITIVE first Lyapunov coefficient, so the Hopf bifurcation is SUBCRITICAL and coexists with a large-amplitude, globally attracting limit cycle (hard onset of oscillations with bistability). Results identify saturation of the feedback kinetics as the minimal ingredient required for oscillatory behaviour in this class of models.
Keywords *Hopf bifurcation, first Lyapunov coefficient, limit cycle, Routh–Hurwitz criterion, saturable kinetics, autocatalysis, hydrogen production, nonlinear chemical dynamics.*

The study of the family of quadratic systems with an invariant ellipse via Artés's algorithm

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Artés's algorithm provides a framework for the topological classification of families of low-degree planar polynomial differential systems. It combines three complementary approaches:

- the construction of bifurcation diagrams in the parameter space of suitable normal forms;
- the method of algebraic invariants for polynomial differential systems [1];
- the numerical computation of phase portraits [2].

In this work, we study the family of quadratic differential systems possessing an invariant ellipse, denoted by **QSE.** Necessary and sufficient affine invariant conditions characterizing this family were established in [3].

Applying Artś's algorithm we obtain a complete topological classification of the family **QSE.** Our main result is the following.

Theorem 1. *Consider the family of quadratic systems* ***QSE*** *possessing at least one invariant ellipse, either real or complex. Then this family exhibits exactly* ***40*** *topologically distinct phase portraits. Among them, three possess a limit cycle, which is algebraic and coincides with the invariant ellipse. Furthermore, we derive necessary and sufficient affine invariant conditions that distinguish all detected phase portraits, except for four cases in which the existence of a separatrix connection cannot be decided by algebraic methods alone.*

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Integration of S^p Remotely Almost Periodic Functions

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We study the problem of integration of remotely almost periodic functions (in the sense of Stepanov) defined on the real axis $\mathbb{R} = (-\infty, +\infty)$ with the values from a Banach space $\mathfrak{B}$.

Let $p \in \mathbb{R}$ and $p \geq 1$. Denote by $L^p_{loc}(\mathbb{R}, \mathfrak{B})$ the space of all locally integrable functions $\varphi : \mathbb{R} \to \mathfrak{B}$ equipped with the distance

$$d_p(\varphi, \psi) := \sup_{\ell > 0} \min\{d_{\ell,p}(\varphi, \psi); \frac{1}{\ell}\}, \tag{1}$$

where

$$d_{\ell,p}(\varphi, \psi) := \Big(\int\limits_{[-\ell,\ell]} |\varphi(s) - \psi(s)|^p ds \Big)^{1/p}.$$

Let $(L^p_{loc}(\mathbb{R}, \mathfrak{B}), \mathbb{R}, \sigma)$ be the shift dynamical system on the space $L^p_{loc}(\mathbb{R}, \mathfrak{B})$.

Definition 1. A function $\varphi \in L^p_{loc}(\mathbb{R}, \mathfrak{B})$ is called [1,2] S^p almost periodic (almost periodic in the sense of Stepanov) if for arbitrary $\varepsilon > 0$ the exists a number $l = l(\varepsilon) > 0$ such that on every segment $[a, a + l]$ $(a \in \mathbb{R})$ there exists at least one number τ for which we have

$$\int_0^1 \|\varphi(t + s + \tau) - \varphi(t + s)\|^p ds < \varepsilon^p \tag{2}$$

for all $t \in \mathbb{R}$.

Remark 1. 1. Every almost periodic (in the sense of Bohr) function $\varphi \in C(\mathbb{R}, \mathfrak{B})$ is also S^p almost periodic.

2. The function $\varphi \in C(\mathbb{R}, \mathbb{R})$ defined by

$$\varphi(t) := \sin \frac{1}{2 + \cos t + \cos \sqrt{2}t}$$

for all $t \in \mathbb{R}$ is S^p almost periodic but it is not Bohr almost periodic [2].

Theorem 1 ([1]). *Let $\varphi \in L^p_{loc}(\mathbb{R}, \mathfrak{B})$ be a S^p almost periodic. Assume that the primitive Φ of φ is compact then it is Bohr almost periodic.*

Recall that a function $\varphi \in L^p_{loc}(\mathbb{R}, \mathfrak{B})$ is said to be

1. S^p asymptotically almost periodic [3] if there are $p, r \in C(\mathbb{R}, \mathfrak{B})$ such that $\varphi = p + r$, p is S^p almost periodic and
$$\int_0^1 \|r(t+s)\|^p ds \to 0$$
as $t \to +\infty$;

2. S^p remotely almost periodic [4] if for every $\varepsilon > 0$ there exists a positive number $l = l(\varepsilon)$ such that on every segment $[a, a+l]$ $(a \in \mathbb{R})$ there exists at least one number τ and $L(\varepsilon, \tau) > 0$ so that
$$\int_0^1 \|\varphi(t+s+\tau) - \varphi(t+s)\|^p ds < \varepsilon^p$$
for all $t \geq L(\varepsilon, \tau)$.

Remark 2. 1. Every S^p almost periodic function is S^p asymptotically almost periodic.

2. Every S^p asymptotically almost periodic function is S^p remotely almost periodic.

3. The converse statements are false.

A natural question arises:

Problem 1. Is there an analog of Theorem 1 for S^p remotely almost periodic functions?

The following theorem holds.

Theorem 2 ([5]). *Let $\varphi \in L^p_{loc}(\mathbb{R}, \mathfrak{B})$ be a S^p remotely almost periodic function and Φ be its primitive. Assume that the following conditions are fulfilled:*

1. *the function φ is S^p translation compact (i.e., the set $\{\varphi^h |\ h \in \mathbb{R}\}$ is a pre-compact subset of $L^p_{loc}(\mathbb{R}, \mathfrak{B})$);*

2. *the set ω_φ is a minimal is a minimal subset of the shift dynamical system $(L^p_{loc}(\mathbb{R}, \mathfrak{B}), \mathbb{T}, \sigma)$;*

3. *the set $\overline{\Phi(\mathbb{R})}$ is compact.*

Then the primitive Φ is remotely almost periodic.

Finally, we note that in general case (the set ω_φ is not minimal) the **Problem 1** remains open.

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Approximation of linear systems with delay, their quasi-polynomials and numerical simulation

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In this paper, we consider a scheme for approximating the solutions of the initial value problem for linear differential-difference equations and their quasi-polynomials by a sequence of special Cauchy problems for a system of ordinary differential equations.

Consider the initial value problem for a linear system of differential equations with delay

$$\frac{dx}{dt} = A_0 x(t) + \sum_{i=1}^{k} A_i\, x(t-\tau_i), \quad t \in [0,T], \tag{1}$$

$$x(t) = \varphi(t), \quad t \in [-\tau, 0], \tag{2}$$

where A_i, $i = \overline{0,k}$ are fixed $n \times n$ matrices, $x \in \mathbb{R}^n$, $0 < \tau_1 < \cdots < \tau_k = \tau$, $\varphi(t) \in C[-\tau, 0]$.

Let's associate with the original problem (1) – (2) the following Cauchy problem for a system of ordinary differential equations [1, 2]:

$$\frac{dz_0(t)}{dt} = A_0 z_0(t) + \sum_{i=1}^{k} A_i\, z_{l_i}(t), \quad l_i = \left[\frac{\tau_i m}{\tau}\right], \tag{3}$$

$$\frac{dz_j(t)}{dt} = \mu\big(z_{j-1}(t) - z_j(t)\big), \quad j = \overline{1,m}, \quad \mu = \frac{m}{\tau}, \quad m \in \mathbb{N},$$

$$z_j(0) = \varphi\left(-\frac{\tau j}{m}\right), \quad j = \overline{0,m}. \tag{4}$$

The connection between the solution of the Cauchy problem (3) – (4) and the original problem (1) – (2) is established by the following result.

Theorem 1. *The solutions of the Cauchy problem (3) – (4) approximate the solution of the initial-value problem (1) – (2), and the following convergence holds:*

$$\left\| x\left(t - \frac{\tau j}{m}\right) - z_j(t) \right\| \to 0, \quad j = \overline{0,m}, \quad t \in [0,T], \quad m \to \infty. \tag{5}$$

Lemma 1. *For the characteristic equation of the approximating system (3), the following expression holds*

$$\Psi_m(\lambda) = \det\left(\lambda E - A_0 - \sum_{i=1}^{k} A_i \left(\frac{\mu}{\mu+\lambda}\right)^{l_i}\right)(\mu+\lambda)^{mn}. \tag{6}$$

For a fixed $\lambda \in \mathbb{N}$ the sequence of functions (6) converges to the quasi-polynomial $D(\lambda) = \det\left(\lambda E - A_0 - \sum_{i=1}^{k} A_i\, e^{-\lambda\tau_i}\right) = 0$ as $m \to \infty$.

According to this lemma, to approximate the roots of the quasi-polynomial, it is sufficient to consider the roots of the characteristic polynomial $\Psi_m(\lambda)$. Based on the considered approximation schemes, modeling of systems with delay [3] was carried out for test model examples.

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Algebraic limit cycles in cubic differential systems with a weak focus

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We consider the cubic system of differential equations

$$\begin{cases} \dot{x} = y + p_2(x,y) + p_3(x,y) \equiv P(x,y), \\ \dot{y} = -x + q_2(x,y) + q_3(x,y) \equiv Q(x,y), \end{cases} \tag{1}$$

where $p_j(x,y)$, $q_j(x,y) \in \mathbb{R}[x,y]$, $j \in \{1,2\}$ are homogeneous polynomials of degree j, and the vector field $X = P\frac{\partial}{\partial x} + Q\frac{\partial}{\partial y}$ associated to system (1). The origin $O(0,0)$ is a singular point for (1) with purely imaginary eigenvalues ($\lambda_{1,2} = \pm i$), i.e. a weak focus [1]. In this paper we investigate cubic differential systems (1) with a weak focus having invariant algebraic curves and algebraic limit cycles.

An algebraic curve $f(x,y) = 0$, $f \in \mathbb{C}[x,y]$ is called invariant algebraic curve of the vector field X if there exists a polynomial $K(x,y) \in \mathbb{C}[x,y]$ such that

$$X(f) = f(x,y)K(x,y).$$

An algebraic limit cycle of degree d of $\mathbb{X}$ is an oval of an real irreducible invariant algebraic curve $f = 0$ of degree d which is a limit cycle of $\mathbb{X}$ [2], [3].

We show that the presence of a singular point the weak focus in cubic differential systems imposes restrictions on the number of degrees of the algebraic limit cycles [1], [6]. We construct

examples of cubic differential systems with a weak focus having invariant algebraic curves and algebraic limit cycles [4], [5].

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Analytical Kamenev-Type Oscillation Criteria with Numerical Illustration for Higher-Order Delay Differential Equations

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In this paper, Kamenev-type oscillation criteria are investigated for a class of higher- order delay differential equations with damping. By employing a generalized Riccati transformation, several sufficient conditions for oscillation are established, which are either new or improved upon recent results available in the literature. Although symmetry considerations are often implicit in oscillation studies, they play a crucial role in guiding the analytical approach and in identifying meaningful directions for further research. In addition, a numerical illustration is presented to visually demonstrate the oscillatory behavior predicted by the theoretical criteria and to support the applicability of the obtained results.

Keywords *Oscillation criteria; Higher-order delay differential equations; Kamenev-type conditions; Generalized Riccati technique; Numerical illustration.*

Jacobi Stability and Kosambi-Cartan-Chern (KCC) Geometric Theory for Chaotic Systems

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Jacobi stability and the Kosambi-Cartan-Chern (KCC) theory provide a powerful differential-geometric framework to analyze chaotic systems. Instead of relying purely on traditional Lyapunov exponents, KCC theory transforms systems of first-order differential equations into a set of second-order equations, using five intrinsic geometric invariants to map the exact pathways of trajectory deviations over time. The KCC geometric theory of stability involves translating a chaotic dynamical system into a system of second-order differential equations (SODE) on a tangent bundle. This allows researchers to study the motion as if it were a trajectory on a curved manifold, driven by physical-like forces and geometric curvatures rather than just abstract variables.
In this work, a three dimensional quadratic polynomial autonomous smooth chaotic system is considered. By reformulating the first-order differential system as a system with two second-order differential equations, we will investigate the nonlinear dynamics of the system from the Jacobi stability perspective through the Kosambi-Cartan-Chern (KCC) geometric theory. The intrinsic geometric properties of the systems will be studied by determining the associated geometric objects: the zero-connection curvature tensor, the nonlinear connection, the Berwald connection, and the five KCC invariants (the external force - the first invariant; the deviation curvature tensor - the second invariant; the torsion tensor - the third invariant; the Riemann–Christoffel curvature tensor - the fourth invariant; the Douglas tensor - the fifth invariant). In order to obtain necessary and sufficient conditions for the Jacobi stability near each equilibrium point, the deviation curvature tensor will be determined at each equilibrium point. Finally, to delve deeper into the dynamics of this chaotic system, we will compare Jacobi stability with classical (linear) Lyapunov stability.

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Keywords *chaotic system, Jacobi stability, KCC-theory*

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On the functional basis of invariant elements for the differential system $s^3(1,3)$

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We will examine the differential system $s^3(1,3)$ in the tensorial form

$$\frac{dx^j}{dt} = a^j_\alpha x^\alpha + a^j_{\alpha\beta\gamma} x^\alpha x^\beta x^\gamma \ (j,\alpha,\beta,\gamma = \overline{1,3}), \tag{1}$$

where the coefficient tensor $a^j_{\alpha\beta\gamma}$ is symmetrical in lower indices in which the complete convolution holds.

In [1] was determined the number of the invariant center-affine polynomials of the system (1).

Nevertheless, the problem of constructing functional bases for invariants, comitants, contravariants, and mixed comitants (see, for example [2–8]) remains open for ternary differential systems.

Using Aronhold's symbolic [10] were constructed an irreducible invariants $\mathcal{J}_{1,3}-\mathcal{J}_{34,3}$, comitants $\mathcal{K}_{1,3}-\mathcal{K}_{18,3}$, contravariants $\mathcal{R}_{1,3}-\mathcal{R}_{14,3}$ and mixed comitants $\mathcal{S}_{1,3}-\mathcal{S}_{21,3}$ for ternary differential system $s^3(1,3)$.

Theorem 1. 1. *The invariants* $\mathcal{J}_{1,3} - \mathcal{J}_{3,3}$, $\mathcal{J}_{7,3} - \mathcal{J}_{19,3}$, $\mathcal{J}_{20,3} - \mathcal{J}_{34,3}$ *forms a functional basis of the irreducible invariants of the ternary differential system* $s^3(1,3)$;

2. *The invariants* $\mathcal{J}_{2,3}$, $\mathcal{J}_{3,3}$, $\mathcal{J}_{7,3} - \mathcal{J}_{20,3}$ *together with the comitants* $\mathcal{K}_{1,3} - \mathcal{K}_{18,3}$ *forms a functional basis of the irreducible comitants of the ternary differential system* $s^3(1,3)$;

3. *The invariants* $\mathcal{J}_{1,3} - \mathcal{J}_{20,3}$ *together with the contravariants* $\mathcal{R}_{1,3} - \mathcal{R}_{14,3}$ *forms a functional basis of the irreducible contravariants of the ternary differential system* $s^3(1,3)$;

4. *The comitants* $\mathcal{K}_{4,3}$, $\mathcal{K}_{8,3}$ *together with the contravariants* $\mathcal{R}_{1,3} - \mathcal{R}_{14,3}$, *mixed comitants* $\mathcal{S}_{1,3} - \mathcal{S}_{21,3}$ *forms a functional basis of the irreducible mixed comitants of the ternary differential system* $s^3(1,3)$.

Note 1. *It is obvious that the set* $\mathcal{J}_{1,3}-\mathcal{J}_{34,3}$, $\mathcal{K}_{1,3}-\mathcal{K}_{18,3}$, $\mathcal{R}_{1,3}-\mathcal{R}_{14,3}$, $\mathcal{S}_{1,3}-\mathcal{S}_{21,3}$ *may contain other functional bases of centro-affine invariants, comitants, contravariants and mixed comitants for the systems* $s^3(1,3)$ *other than those given in the Theorem* 1.

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On the number of functionally independent polynomial $SO(2,\mathbb{R})$-invariants of the differential system $s(1, m_1, m_2, ..., m_l)$ involved in solving the stability problem of unperturbed motion in the case of a center and a focus

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Consider the two-dimensional polynomial system $s(1, m_1, m_2, ..., m_l)$ consisting of differential equations of perturbed motion [1, §1.2], of the form:

$$\frac{dx}{dt} = y + \sum_{i=1}^{l} P_{m_i}(x, y), \quad \frac{dy}{dt} = -x + \sum_{i=1}^{l} Q_{m_i}(x, y) \tag{1}$$

where P_{m_i} and Q_{m_i} are real homogeneous polynomials of degree $m_i > 1$. The set $\{1, m_1, m_2, ..., m_l\}$ consists of a finite number ($l < \infty$) of distinct natural numbers. It is known [2] that, for the system (1), the origin is a singular point of the second group (center or focus).

Let us denote by

$$N = 2\sum_{i=1}^{l}(m_i - 1) \tag{2}$$

the maximum number of all possible non-zero coefficients of the nonlinear right-hand side of the system (1).

Using the Lie operator of the linear representation of the rotation group $SO(2,\mathbb{R})$ in the space of phase variables and coefficients of the system (1) [3], the following theorem has been proven

Theorem 1. *The upper bound of the number of functionally independent $SO(2,\mathbb{R})$ -semi-invariants ($SO(2,\mathbb{R})$ -invariants) [3] involved in solving the stability problem of unperturbed motion [1] governed by system (1) does not exceed $N + 1$ ($N - 1$), where N is given by (2).*

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The study of the integrability of cubic differential systems with six invariant straight lines of two directions through analytical and software methods

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The study focuses on a specific class of planar cubic differential systems that admit six invariant straight lines (along with their parallel multiplicity) grouped in two directions. Thus, by applying affine coordinate transformations and time scaling, the studied class can be reduced to exactly 12 unique canonical forms. For each of the 12 differential systems, the form of the first integral is explicitly determined. The forms of the integrals vary from purely rational/algebraic structures to highly complex forms containing inverse trigonometric or exponential functions. The problem of integrability was solved using the Darboux method, the direct method and the software method.

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Qualitative analysis of planar polynomial differential systems up to degree six with the line at infinity of maximal multiplicity

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We study planar polynomial differential systems of degree $n \leq 6$:

$$\begin{cases} \dot{x} = \sum_{k=0}^{n} \sum_{i=0}^{k} a_{i,k-i}\, x^i y^{k-i}, \\ \dot{y} = \sum_{k=0}^{n} \sum_{i=0}^{k} b_{i,k-i}\, x^i y^{k-i}, \end{cases} \tag{1}$$

with real coefficients $a_{i,j}, b_{i,j}$.
The problem of determining the maximal multiplicity of the invariant line at infinity, denoted by $M_\infty(n)$, and studying the corresponding dynamics has been investigated for systems of lower degrees by several authors, including A. Şubă [3], O. Vacaraş [4] and others. Under the framework of the conjecture $M_\infty(n) = 3n - 2$ for any degree $n \geq 2$, the canonical forms are presented and the topological phase portraits on the Poincaré disk are fully classified for degrees $n = 1, 2, 3, 4, 5$, where the maximal multiplicities are $M_\infty(1) = 1$, $M_\infty(2) = 4$, $M_\infty(3) = 7$, $M_\infty(4) = 10$, and $M_\infty(5) = 13$, respectively. Furthermore, we aim to obtain the qualitative behavior and construct the topological phase portraits on the Poincaré disk for the specific degree-six ($n = 6$) polynomial differential system presented in [2], whose invariant line at infinity reaches the multiplicity 16.

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Dynamics of Some Delayed Nutrient Recycling Models

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Starting from a general mathematical model describing a closed nutrient–phytoplankton–zooplankton ecosystem with delayed nutrient recycling, two models are derived by approximating the distributed delay with gamma distribution functions having one and two degrees of freedom. The resulting formulations are described by systems of ordinary differential equations of four and five dimensions, respectively. The aim of the study is to investigate the influence of the mean recycling delay and the total nutrient content on the stability and long-term dynamics of the ecosystem. Using local stability and bifurcation theory, the equilibrium solutions are characterized and their stability is analyzed. It is shown that both models display qualitatively similar dynamics: each admits at most three equilibrium points, with at most one being locally asymptotically stable, and stability is exchanged through transcritical bifurcations. Under suitable assumptions on the functional response, the interior nutrient–phytoplankton–zooplankton equilibrium loses stability through a supercritical Hopf bifurcation, giving rise to a stable limit cycle. The results obtained for the reduced four-dimensional model are compared with those of the corresponding five-dimensional model, demonstrating that the reduction preserves the essential dynamical features. The findings are discussed in the context of previous studies, highlighting both their consistency with earlier results and the new insights provided by the reduced formulation, together with their implications for ecosystem dynamics.

Quartic differential systems with a center-focus critical point, two parallel invariant straight lines, and the line at infinity of maximal multiplicity

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We consider a real quartic differential system having a center-focus critical point. By an affine transformation of coordinates and time rescaling, the system can be brought to the form

$$\begin{cases} \dot{x} = y + p_2(x,y) + p_3(x,y) + p_4(x,y) \equiv P(x,y), \\ \dot{y} = -(x + q_2(x,y) + q_3(x,y) + q_4(x,y)) \equiv Q(x,y), \\ \gcd(P,Q) = 1,\ yp_4(x,y) + xq_4(x,y) \not\equiv 0, \end{cases} \tag{1}$$

where $p_i(x,y)$ and $q_i(x,y)$, $i \in \{2,3,4\}$, are homogeneous polynomials in x and y of degree i with real coefficients.

In this paper, we determine the maximal multiplicity of the line at infinity for the quartic system (1) assuming that the system has two parallel invariant straight lines. It was proved the following theorem.

Theorem 1. *In the class of quartic differential systems with a center-focus critical point, non-degenerate infinity, and two parallel invariant straight lines, the maximal multiplicity of the line at infinity is equal to five.*

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3. Mathematical Modeling

Soil erosion model in ASTERIX. Part II: Numerical simulations.

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ASTERIX is a software package based on an integrated mathematical model that combines an extended Saint-Venant system of equations for water flow with a Hairsine-Rose-type model for soil erosion on vegetated surfaces. This work aims to validate both the continuous and discrete models through a series of numerical experiments.

The first series of simulations considers flash floods generated by a dam-break event in a long rectangular flume partially covered by vegetation. These tests highlight the influence of vegetation cover on soil erosion. To further investigate this effect, a simulation of a real hydrographic basin is also presented.

Finally, numerical solutions are compared with analytical solutions for selected configurations of the soil surface and vegetation density distribution.

Acknowledgments. This work was carried out within the framework of the "Analysis and Control of Differential Systems" research program of the Romanian Academy.

Two-dimensional systems with symmetry / anti-symmetry

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Certain two-dimensional mathematical systems can also be analysed from the point of view of the physical significance they may have within interdisciplinary analogies. Transforming two arbitrary variables into two others, one symmetric (even) and the other anti-symmetric (odd), highlights the interpretative symmetry properties of the mathematically described physical system. Thus, the concept of a differential amplifier in electronics can also be extended to the thermodynamics of irreversible processes. The spin space in quantum physics, being two-dimensional, has a dynamics compatible with information theory (similarly to polarised light). The problem of parity of the wave function is analysed in connection with its close relationship to wave-particle duality.
Keywords: symmetrical / anti-symmetrical, differential amplifier, wave function parity.

Approaching the inverse problem modelling for the Chua dynamical system in a simplified case

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Inverse problems are strongly connected with various branches of science and technology. In inverse modelling, the study relies on three main concepts: measurement operator (which is basically related to data and parameters), noise model (related to perturbations), and prior model. The relations between them form an analysis panel, specific for each concrete problem. The present paper focuses on approaching the inverse modelling problem for the Chua circuit model, in a simplified case. The condition number of a matrix is taken into account, since this tool is easy to apply especially for small dimension systems. The aim is to gain information about the system's parameters from the model's matrix, and to test this technique for the system's stability.
AMS Subject Classification (2010): 37C75, 34A55, 65L09, 65F18
Keywords *Dynamical system, Chua circuit, Inverse problem, Numeric visualisation*

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Soil erosion model in ASTERIX. Part I: some mathematical properties and analytic solutions

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The mathematical model considered in this work consists of an extended Saint-Venant system of equations for water flow and a Hairsine-Rose-type model for erosion processes on plant-covered soil surfaces. We briefly describe the numerical approximation scheme associated to the continuous non-linear hyperbolic system of PDEs implemented in ASTERIX software.

Despite the complexity of the model, there are some configurations of the soil surface and vegetation density distribution that allow us to obtain analytical solutions. One such solution is

found for the steady-state regime of flow and erosion. For this case, we show that the equilibrium state of the numerical solution converges to the stationary solution of the continuous system. Moreover, it is shown that this numerical equilibrium solution is a stable state.

For the non-stationary case, some analytic solutions are presented for the case of a single-class sediment.

Acknowledgments. This work was carried out within the framework of the "Analysis and Control of Differential Systems" research program of the Romanian Academy.

An Electrochemical Corrosion Problem in Oil Storage Tanks

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One of the most common problems in the petroleum industry concerns the protection of storage tanks against electrochemical corrosion. While oil itself is not a good conductor of ions, the water that inevitably accompanies it and accumulates at the bottom of these storage tanks allows ionic conduction, and produces corrosion.

The paper analyzes this problem for the roof oil tanks of ANCAP (the Uruguayan state-owned company that refines, distributes, and sells petroleum products). The mathematical model proposed consists in a parabolic PDE, which after several simplifications results in an ODE with variable coefficients.

The mathematical tool utilized to solve the ODE is the Laplace Transform, which gives an elegant solution based on modified Bessel functions. This final result improves previous solutions in the form of power series, that presented greater problems in their practical application.

Keywords *Laplace transform, Bessel functions, electrochemical corrosion, oil tanks.*

On the generalization of the Lanchester recursive model

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The article deals with the Lanchester recursive quadratic model [15], where a generalization is required, in the sense that the ratio of the forces involved in the combat action, respectively the ratio of their effective degradation rates, are real, positive, non-zero numbers. Each time, for p-combat engagements, $p \in \mathbb{N}^*$, we obtain predictions for the ratio of forces and annihilation and combat times, respectively.

2020 Mathematics Subject Classification: Primary 34C60, 34K60; Secondary 65L05, 65S05

Keywords *Lanchester combat models, recursive combat model, annihilation and combat times, generalization*

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Modeling, simulation, and implementation of concurrent computing based on membrane p-system architectures

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This paper presents the modelling, simulation, and implementation of concurrent computing based on membrane P-system architectures, employing the principles of membrane computing to model and execute parallel processes in distributed computing environments. The proposed system integrates genetic algorithms, a Master–Worker architecture, Docker containers, and Redis Pub/Sub mechanisms to establish a virtual computing network capable of distributing computational tasks among independent processing nodes. The modelling stage defines membranes, objects, and evolution rules; the simulation stage reproduces the concurrent behaviour of the system; and the implementation stage enables practical execution, performance evaluation, and real-time monitoring through dashboards and benchmarking tools. Experimental results demonstrate reduced execution time, improved scalability, and the potential for extending the proposed architecture toward reconfigurable computing platforms and virtual machines suitable for distributed optimisation and high-performance concurrent computing.

Keywords *membrane computing, P systems, concurrent computing, distributed computing architecture, reconfigurable computing, virtual machine, virtual computing network, computing cluster, distributed optimisation, genetic algorithms.*

Pulsatory liposome and artificial neuronal network

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We consider an unilamellar liposome (lipid vesicle) filled with aqueous solution of osmotic solute. Due to the osmosis process the liposome swells to a critical size, when a transbilayer pore suddenly appears. Some of the internal solution leaks through this pore and the liposome relaxes and returns to its initial size. The swelling starts again and the liposome begins a new cycle and so on. The evolution of the liposome is a dynamic and cyclical process. All the processes which contribute to the liposome relaxing and its coming back to the initial size (pore evolution and internal solution delivery) are described by three differential equations. This system of differential equations describes the evolution of the pulsatory liposome during a cycle and can be integrated using numerical and analytical [1,2] methods. The liposome can be assimilated to a intelligent biophysical engine and a neural network. Each neuronal layer learns to solve the equations that describe the evolution of the liposome during a cycle.

Keywords *pulsatory liposome, biophysical engine, neural network.*

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4. Real, Complex, Functional and Numerical Analysis

Homogenization Results for a Diffusion Problem in a Composite Material with a Rough Imperfect Interface

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In this talk, we discuss several homogenization results for a stationary heat diffusion problem set in a domain made up of two connected components separated by a highly oscillating imperfect interface, whose geometry changes periodically at a scale given by a small parameter ε. We suppose that the solution of our problem is continuous across the interface, where the jump of its co-normal flux is driven by an oscillating source G^ε, modulated by a factor depending on ε. The analyzed problem also involves two parameters, α and β, which are associated to the amplitude of the oscillations of the rough interface and to the magnitude of the source term, respectively. Our aim is to understand the interplay between the micro-oscillations of the interface and the imperfect transmission conditions and to offer a precise description of the resulting effective medium. The values of the two model parameters α and β strongly influence the homogenization process, leading to distinct macroscopic limit problems.

Finite Element Analysis of MHD Natural Convection and Entropy Generation in an Al_2O_3–Water Nanofluid-Filled Corrugated Porous Annulus

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Efficient control of heat transfer in porous and magnetically influenced fluid systems has become a key challenge in modern thermal engineering applications. This study numerically examines the behavior of Al_2O_3–water nanofluid flow and heat transfer within a geometrically complex porous annulus subjected to a uniform magnetic field. The configuration includes a corrugated inner heated cylinder, which introduces additional flow disturbances and enhances thermal mixing. The governing transport equations are formulated using the Brinkman–Forchheimer porous medium model along with magnetohydrodynamic effects and are solved using a finite element approach. Emphasis is placed on understanding how flow structure and temperature distribution respond to variations in buoyancy strength, magnetic field intensity, nanoparticle concentration, porosity, and surface corrugation. The results show that stronger buoyancy forces significantly enhance circulation strength and heat transport, while magnetic effects suppress fluid motion and weaken convective activity. The inclusion of nanoparticles improves overall thermal response due to increased effective conductivity, whereas higher porosity reduces flow resistance and promotes stronger convection. Additionally, geometric corrugation of the heated surface intensifies mixing by disrupting boundary layers, leading to improved thermal performance. Entropy generation analysis highlights the competition between thermal and fluid friction irreversibility under different physical regimes. The study demonstrates how combined control of magnetic field, porous structure, and nanofluid properties can be used to optimize heat transfer systems involving complex geometries.

Sharp numerical approximation of the fractional Sobolev constant

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We establish sharp estimates for the optimal constant of the fractional Sobolev inequality in dimension $N \geq 1$, with fractional exponent $s \in \left(0, \min\left\{1, \frac{N}{2}\right\}\right)$,

$$[u]^2_{\dot{H}^s(\mathbb{R}^N)} \geq S_{N,s} \|u\|^2_{L^{2^*_s}(\mathbb{R}^N)},$$

where

$$[u]^2_{\dot{H}^s(\mathbb{R}^N)} := s(1-s) \int_{\mathbb{R}^N} \int_{\mathbb{R}^N} \frac{|u(x)-u(y)|^2}{|x-y|^{N+2s}} \, \mathrm{d}x \, \mathrm{d}y$$

is the Gagliardo seminorm of the function $u \in C_c^\infty(\mathbb{R}^N)$,

$$2^*_s := \frac{2N}{N-2s},$$

and

$$S_{N,s} = 2s(1-s) \left(\int_{\mathbb{R}^N} \frac{1-\cos(\zeta_1)}{|\zeta|^{N+2s}} \mathrm{d}\zeta \right) 2^{2s} \pi^s \frac{\Gamma\left(\frac{N+2s}{2}\right)}{\Gamma\left(\frac{N-2s}{2}\right)} \left(\frac{\Gamma\left(\frac{N}{2}\right)}{\Gamma(N)} \right)^{\frac{2s}{N}}$$

is the optimal constant.

The convergence rates that we establish take place for the Galerkin approximation with piecewise linear elements, when the computations are carried out in the unit ball, for which we employ a quasi-uniform and regular mesh and they read as follows:

$$S_{N,s,h} - S_{N,s} \sim h^{\alpha_{N,s}},$$

where

$$\alpha_{N,s} = \frac{2(2-s)(N-2s)}{N+4(1-s)}.$$

MSC 2020 65N30, 46E35
Keywords *Fractional Sobolev Inequalities, Approximation and Stability, Finite Element Method, Gagliardo-Nirenberg Interpolation Inequalities*

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Homogenization of the dissipative system for the wave equation in a perforated periodic domain

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The presented work is the homogenization of a dissipation system for the wave equation in a perforated periodic domain. The novelty of the article lies in the homogenization of such a system, i.e. the appearance of the term $\frac{\partial u_\varepsilon}{\partial t}$ in the boundary condition on the boundary of the holes of the perforated domain. For homogenization, we apply the method of multiple scales [5], and for the convergence of the homogenization process we use the double-scale convergence of [1] and [2].

The result of homogenization is an equation of waves on the homogeneous domain (without holes). The mathematical model in practice can be associated with the dissipation of vibrations in a field where identical holes are periodically distributed. The homogenized wave equation provides the amplitude and frequency of the dissipated waves.

Applications of Fractional Calculus in Geometric Function Theory

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This presentation focuses on the introduction of fractional integral operators and their investigation from different perspectives within geometric function theory.

The first major perspective focuses on establishing conditions for univalence in the open unit disk. By investigating a fractional integral operator involving the Bessel function of the first kind, criteria for univalence, starlikeness, and convexity are derived using differential subordination methods and inclusion properties [1]. Similarly, sufficient univalence conditions are established through the technique of subordination chains applied to the fractional integral of the confluent (Kummer) hypergeometric function [2]. The second perspective applies these operators to generalized subordination theories. We analyze their application in obtaining new fuzzy differential subordinations [3], alongside recent extensions into third-order strong differential subordinations centered around the fractional integral, utilizing both confluent and Gaussian hypergeometric functions [4].

These combined results highlight how the introduction and investigation of fractional operators generate extensive possibilities for their utilization across geometric function theory. As a concrete development, the geometric properties established for the first fractional integral operator [1] led to the definition and univalence study of a new integral operator [5]. Due to these properties, this new operator can be applied in future studies and opens new directions for research.

Keywords*Fractional integral operators; Univalence conditions; Differential subordinations; Third-order strong differential subordinations; Fuzzy differential subordinations.*
MSC: 30C45, 30C80, 33C10.

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Double-scale homogenization for heat transmission by temperature-jumping conduction in a periodic composite

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The problem presented in the article refers to the heat transfer between the components of a composite made of two different materials and a periodic thermal barrier through which the heat flux has a term proportional to the temperature jump - in the sense that the jump is multiplied by the resistivity of the material from which the barrier is composed. This mathematical model is solved with the help of homogenization theory, more precisely with the double scale convergence method and the multiple scale method. Thus, from a heat problem on a composite domain ε-periodic one arrives at a heat problem on a homogeneous domain, containing conduction vectors and the mean of the divergence of the corrector gradient.

A weak demi version of Dunford-Pettis operators

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Let X and Y be Banach spaces and X', Y' be norm dual of X, Y, respectively. An operator $T : X \to Y$ is called a Dunford-Pettis operator if $x_n \to 0$ weakly in X implies $\| Tx_n \| \to 0$ in Y. Adjoint operator of $T : X \to Y$ is defined by $T' : Y' \to X'$, $(T'f)(x) = f(Tx)$ for every $f \in Y', x \in X$. An operator $T : X \to X$ is said to be demi Dunford-Pettis operator if, for every sequence (x_n) in X such that $x_n \to 0$ weakly in X with $\|x_n - Tx_n\| \to 0$,then $\|x_n\| \to 0$ in X.

Every Dunford-Pettis operator is a demi Dunford-Pettis operator. An operator $T : X \to Y$ is called a weakly Dunford-Pettis operator if, for every sequence (x_n) in X with $x_n \to 0$ weakly in X and (f_n) in Y' with $f_n \to 0$ weakly in Y' , then $f_n(Tx_n) \to 0$ as $n \to \infty$. We define weakly demi Dunford-Pettis operator that it is different from defined by J. Riberio and F. Santos. We show that the results obtained by J. Riberio and F. Santos are also true by using our definition.

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Boundary variation method for the generalized Cheeger problem

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In this talk we study the generalized Cheeger problem to find the safety factor and the collapse domain (onset flow region). We propose a numerical method, based on a boundary variation method. We also present numerical examples in two and three dimensions.

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Applications of the Hilbert metric in Geometric Function Theory

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Hilbert geometry, a generalization of Cayley-Klein's model of hyperbolic space to arbitrary bounded convex domains in $\mathbb{R}^n$, is an active area of research, that can be approached from several perspectives, including calculus of variations, projective geometry, Finsler geometry, computational geometry and dynamical systems [4], [3]. In this talk, we discuss connections between the Hilbert metric and hyperbolic-type metrics, such as the Poincaré metric, the distance ratio metric and the visual angle metric. We present recent results obtained by Matti Vuorinen and some of his collaborators, regarding comparisons between the Hilbert metric and various hyperbolic-type metrics, ball inclusion relations and distortion of the Hilbert metric under quasiregular mappings [5], [1], [2].

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On direct methods in solving of some classes of linear and nonlinear singular integral equations

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In this paper the main stages of development of the theory of direct approximate methods in solving singular integral equations is described. We suggest the computational algorithms of collocation, quadratures and reduction methods for the approximate solving of nonlinear singular integral equations and of mixed linear singular integral equations. These equations are defined on the arbitrary smooth closed contour Γ of complex plane. The collocation and quadratures methods are based on the Fejer knots. The reduction method is based in using the Fourier polynomials on the unit circle Γ_0 and Faber-Laurent polynomials on the arbitrary smooth closed contour Γ.
Theoretical justification of direct approximate methods in solving the nonlinear singular integral equations has been obtained in Hölder, generalized Hölder and Lebesgue spaces. In the case of more smooth contours Γ, namely contours from the class $C(2,\gamma)$, the unimprovable estimations for the rate of convergence of approximate solutions for quadratures method are obtained.
For mixed linear singular integral equations defined on arbitrary smooth closed contour Γ, the theoretical justification in Hölder and Lebesgue spaces of collocation and quadratures methods in solving, has been obtained. For reduction method the theoretical justification in Hölder spaces for these equations are also obtained.
The efficiency of approximate methods is determined by its concrete algorithmically implementation, by it programs, flexible integrated in Intelligent Support System (ISS), having the purpose to serve more successful the user. Algorithmically implementation of approximate methods, theoretically justified in diverse Banach spaces, in solving of regular IE, weakly-singular and specially singular equations, defined on real and complex domains, is subordinated to some general principles in application of ISS for IE, which are described.
For example, implementation of approximate methods for solving weakly singular integral equations (WSIE) of second kind leads to the necessity of concrete estimation in Banach spaces of Fredholm and Volterra weakly singular integral operators (WSIO) and their modifications that

appear in proposed algorithms.
Concrete values of the constants that estimate the Fredholm and Volterra WSIO in the spaces of continuous function and Hölder spaces, are obtained. Then, based on the obtained results, we give theoretical foundation in the spaces of continuous function and the Hölder spaces for collocation, quadratures, splin-collocation and splin-quadratures methods in solving WSIE of second kind.

Minkowski-type integral inequalities for vector functions

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We present some integral inequalities for vector functions for Choquet and Sugeno type integrals

Solution Approximation of Nonautonomous Lattice Dynamical Systems

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The purpose of this article is to justify the projection method for solving the Cauchy problem for an infinite-dimensional system

$$u_i' = \nu(u_{i-1} - 2u_i + u_{i+1}) - \lambda u_i + F(u_i) + f_i(t) \ (i \in \mathbb{Z}). \tag{1}$$

Denote by $\mathbb{R} := (-\infty, \infty)$, $\mathbb{Z} := \{0, \pm 1, \pm 2, \ldots\}$ and ℓ_2 the Hilbert space of all two-sided sequences $\xi = (\xi_i)_{i\in\mathbb{Z}}$ $(\xi_i \in \mathbb{R})$ with

$$\sum_{i\in\mathbb{Z}} |\xi_i|^2 < +\infty$$

and equipped with the scalar product

$$\langle \xi, \eta \rangle := \sum_{i\in\mathbb{Z}} \xi_i \eta_i.$$

Let $(\mathfrak{B}, \|\cdot\|)$ be a Banach space with the norm $\|\cdot\|_{\mathfrak{B}}$, $C(\mathbb{R}, \mathfrak{B})$ be the space of all continuous functions $f : \mathbb{R} \to \mathfrak{B}$ equipped with the compact-open topology.

We study the finite-dimensional approximations of solutions of the systems (1), where $\lambda > 0$, $F \in C(\mathbb{R}, \mathbb{R})$ and $f \in C(\mathbb{R}, \ell_2)$ $(f(t) := (f_i(t))_{i\in\mathbb{Z}}$ for all $t \in \mathbb{R})$.

The system of equation (1) can be rewritten as follows

$$\dot{v} = -\nu A v - \lambda v - f(v) + \mathfrak{f}(t), \quad t \in \mathbb{T} \quad (\mathbb{T} = \mathbb{R} \text{ or } \mathbb{R}_+) \tag{2}$$

$$v(0) = v_0 \in \ell_2 \tag{3}$$

where A is the discrete Laplacian, $v := (v_i)_{i\in\mathbb{Z}}$, $f(v) := (f(v_i))_{i\in\mathbb{Z}}$ and $\mathfrak{f}(t) := (\mathfrak{f}_i(t))_{i\in\mathbb{Z}}$.

The system (1) can be considered as a discrete analogue of a reaction-diffusion equation in $\mathbb{R}$:

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial^2 x} - \lambda u + F(u) + f(t, x),$$

where grid points are spaced h distance apart and $\nu = D/h^2$.

Below we consider the finite-dimensional approximations of the infinite-dimensional system (1).

Let $\mathfrak{f} \in C(\mathbb{R}, \ell_2)$ and $\mathfrak{f} = (f_i)_{i\in\mathbb{Z}}$. For every $n \in \mathbb{N}$ consider the $2n+1$-dimensional system

$$\begin{cases} \dot{v}_{-n} = -\nu(-v_n + 2v_{-n} - v_{-n+1}) - \lambda v_{-n} - F(v_{-n}) + f_{-n}(t) \\ \dot{v}_{-n+1} = -\nu(-v_{-n} + 2v_{-n+1} - v_{-n+2}) - \lambda v_{-n+1} - F(v_{-n+1}) + f_{-n+1}(t) \\ \vdots \\ \dot{v}_i = -\nu(-v_{i-1} + 2v_i - v_{i+1}) - \lambda v_i - F(v_i) + f_i(t) \quad (|i| < n-1) \\ \vdots \\ \dot{v}_{n-1} = -\nu(-v_{n-2} + 2v_{n-1} - v_n) - \lambda v_{n-1} - F(v_{n-1}) + f_{n-1}(t) \\ \dot{v}_n = -\nu(-v_{n-1} + 2v_n - v_{-n}) - \lambda v_n - F(v_n) + f_n(t) \end{cases} \tag{4}$$

$(v_{-n}, \ldots, v_n)(0) = (v_{0,-n}, \ldots, v_{0,n}) \in \mathbb{R}^{2n+1}$.

Denote by $[\mathbb{R}^n]$ the space of all linear mapping $A : \mathbb{R}^n \to \mathbb{R}^n$ equipped with the operator norm. We can rewrite this system as follows. Let

$$B_n = \begin{pmatrix} -1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & & \ddots & & \vdots & & \\ 0 & 0 & 0 & 0 & \cdots & -1 & 1 \\ 1 & 0 & 0 & 0 & \cdots & 0 & -1 \end{pmatrix} \in [\mathbb{R}^{2n+1}]$$

and denote by $A_n \in [\mathbb{R}^{2n+1}]$ defined by

$$A_n = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & \cdots & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cdots & -1 & 2 & -1 \\ -1 & 0 & 0 & 0 & 0 & \cdots & 0 & -1 & 2 \end{pmatrix} \in [\mathbb{R}^{2n+1}],$$

then we have

$$A_n = B_n^* B_n = B_n B_n^* \quad (B_n^* \text{ the transpose of } B_n) \text{ for every } n \in \mathbb{Z}_0.$$

Let $\mathfrak{f} \in C(\mathbb{R}, \ell_2)$ and $n \in \mathbb{N}$ denote by $\mathfrak{f}^n = (f_i^n)_{|i|\le n} \in C(\mathbb{R}, \mathbb{R}^{2n+1})$ defined by

$$\mathfrak{f}^n = \begin{cases} \mathfrak{f}_i^n = f_i, \text{ if } |i| \le n-1 \\ \mathfrak{f}_n^n = f_{-n-1} \text{ and } \ f_{-n} = f_{n+1}. \end{cases} \tag{5}$$

The system of equation (1) can be rewritten as follows

$$\dot{v} = -\nu A_n v - \lambda v - f^n(v) + \mathfrak{f}^n(t), \quad t \in \mathbb{T} \quad (\mathbb{T} = \mathbb{R} \text{ or } \mathbb{R}_+) \tag{6}$$

$$v(0) = v_0 \in \mathbb{R}^{2n+1} \tag{7}$$

where $v := (v_i)_{|i|\le n}$, $f^n(v) := (f(v_i))_{|i|\le n}$ and $\mathfrak{f}^n(t) := (\mathfrak{f}_i(t))_{|i|\le n}$

Remark 1. Note that the space $\mathbb{R}^{2n+1}$ is embedded in the space ℓ_2 as follow:

$$u \to \mathcal{I}_n(u) := (\ldots, 0, u_{-n}, u_{-n+1}, \ldots, u_{-1}, u_0, u_1, \ldots, u_{n-1}, u_n, 0, \ldots).$$

Remark 2. The system (2) (respectively (2)) can be rewritten in the form

$$v' = -\nu A v - \lambda v + F(v) + f(t) \tag{8}$$

(respectively,

$$v_n' = -\nu A_n v_n - \lambda v_n + F_n(v) + \mathfrak{f}^n(t)). \tag{9}$$

Denote by $\langle \ell_2, \varphi, (H(f), \mathbb{R}, \sigma)\rangle$ (shortly, φ) the cocycle generated by the equation (2) (respectively, $\langle \mathbb{R}^{2n+1}, \varphi_n, (H(\mathfrak{f}^n), \mathbb{R}, \sigma)\rangle$ (shortly, φ_n) the cocycle generated by the equation (2)).

Let $v \in \ell_2$ and $v_n \in \mathbb{R}^n$ such that $I_n v_n \to v$ as $n \to \infty$.

A natural question arises: Is there the equality

$$\lim_{n\to\infty} \max_{|t|\le L} \|\varphi(t, v, f) - I_n \varphi_n(t, v_n, \mathfrak{f}_n)\| = 0 \quad (\forall\ L > 0) \tag{10}$$

holds?

The main result of this paper gives the positive answer to this question.

5. Probability Theory, Mathematical Statistics, Operations Research

Quality of pseudorandom number generators approached through the lens of Monte Carlo simulation of new probabilistic models in network reliability

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The accuracy of Monte Carlo simulation results depends directly on the quality of the pseudorandom number generators (PRNGs) used. In order to analyze the impact of pseudorandom number generators on the degree of concordance between Monte Carlo simulation results and the theoretical simulated model, the following generators were selected: *Java ThreadLocalRandom, Python secrets*, and the sequence based on the *decimal digits of* π. For this purpose, several new dynamic probabilistic models were introduced, targeting the reliability of dual serial-parallel and parallel-serial networks. The lifetimes of the units in each subsystem are considered independent, identically distributed random variables, uniformly distributed on the set $\{0, 1, \ldots, K\}$. The number M of subsystems and numbers $N_1, N_2, \ldots, N_M$ of units in each subsystem are 0-truncated power series distributed random variables. The theoretical lifetime distributions are compared with the empirical distributions obtained through Monte Carlo simulations performed with different PRNGs. The deviations between the theoretical and empirical results constitute the criterion for assessing generator quality. Examples of such models are presented, in which their validation is accompanied by the statistical processing of simulated data in both their uncensored and censored forms.

Keywords *censored data, discrete uniform distribution, maximum likelihood estimators, Monte Carlo simulation, parallel-serial networks, pseudorandom number generators, reliability, serial-parallel networks*

Estimation of Drifting Hidden Markov Models

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This presentation focuses on the modeling and estimation of time series data with latent structures using Hidden Drifting Markov Models (HDMMs). First, we introduce drifting Markov processes, a nonhomogeneous alternative to classical Markov processes, with specific types of nonhomogeneity. Second, we present a theoretical overview of HDMMs, highlighting their key components, probabilistic assumptions, focusing on several modeling approaches. Then we develop the Expectation–Maximization (EM) algorithm for parameter estimation and present some numerical simulations to illustrate the practical implementation and performance of these techniques.

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Performance Analysis of Distributed Database Systems Using the M/M/c Queueing Model

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This paper investigates the performance of distributed database systems by modeling request processing using the M/M/c queueing model. Distributed database architectures commonly employ multiple identical servers operating in parallel to process concurrent client requests, making the M/M/c model suitable for evaluating their performance. The main contribution consists in the statistical analysis of the influence of the number of servers on key performance indicators, including server utilization, waiting probability, average queue length, average waiting time, and total response time. The theoretical characteristics of the model are validated through Monte Carlo simulation experiments performed under different workload scenarios. The obtained results demonstrate a high level of agreement between analytical and empirical estimates and highlight the impact of horizontal scaling on system performance. The proposed approach provides an effective framework for analyzing distributed database infrastructures and supports the selection of an appropriate number of servers to satisfy performance requirements while ensuring efficient resource utilization.

Keywords *M/M/c, distributed databases, queueing theory, Monte Carlo simulation, performance analysis.*

Extended Network Method for Stochastic Games with Multiple Final Sequences of States

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Similarly as in [2], we consider a cyclic game Γ, defined on the system L with the set of states $V = \{v_1, v_2, \ldots, v_\omega\}$ and the final sequences of states $X^{(\ell)} = (x_1^{(\ell)}, x_2^{(\ell)}, \ldots, x_m^{(\ell)}) \in V^m,\ \ell = \overline{1,r}$. The initial probabilities are $(p^*(v))_{v\in V}$ and the strategy of the player Π_s is $(p^{(s)}(u,v))_{u,v\in V}$, $s = \overline{0,R-1}$.

Next, the extended network method is applied. We consider a copy of each state $v \in V$ for each player Π_s, $s = \overline{0,R-1}$. Therefore, the set of states becomes $U = V \times \{0,1,\ldots,R-1\}$. The state is (v,s) at given moment of time t if the state is v and $t \ mod\ R = s$. For each $\ell \in \{1,2,\ldots,r\}$, the final sequence of states $X^{(\ell)}$ corresponds to the multiple sequences of states

$$X^{(\ell,s)} = ((x_1^{(\ell)}, s), (x_2^{(\ell)}, s \oplus 1), \ldots, (x_m^{(\ell)}, s \oplus (m-1))) \in U^m,\ s = \overline{0,R-1},$$

where $j \oplus k = (j+k)\ mod\ R,\ \forall j,k \in \mathbb{N}$. The new initial probabilities are

$$p^*((v,0)) = p^*(v),\ p^*((v,s)) = 0,\ s = \overline{1,R-1},\ \forall v \in V,$$

and, for $s = \overline{0,R-1}$ and $\forall u,v \in V$, the new transition probabilities are

$$p((u,s),(v,s\oplus 1)) = p^{(s)}(u,v),\ p((u,s),(v,k)) = 0,\ k \neq s \oplus 1.$$

Thus, the game Γ is converted into a system with ωR states and rR final sequences of states $X^{(\ell,s)},\ \ell = \overline{1,r},\ s = \overline{0,R-1}$. According to [1], the distribution of the game duration T is a homogeneous linear recurrence of order $R(\omega + mr)$. The probabilistic analysis of T can be done as described in [1] and the win probabilities of the players can be obtained based on [2].

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On the optimal control of semi-Markov processes

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Let $\{N(t), t \geq 0\}$ be a pure birth process with state space $\{0, 1, 2, \ldots\}$, starting at $N(0) = 0$. Suppose that the time T spent in a given state, instead of being exponentially distributed, has a Weibull distribution with density function $f_T(t) = kt^{k-1}e^{-t^k}$, where k is a positive integer. Then $\{N(t), t \geq 0\}$ is no longer a Markovian process, but rather a semi-Markov process. Nevertheless, the process $X(t) := (N(t), S_{N(t)})$, where $S_{N(t)}$ denotes the elapsed time spent in the current state at time t (that is, the age of the process since the last transition), *is* Markovian. The problem of minimizing the time τ that the process will need to move from $N(0) = 0$ to $N(\tau) = j \in \mathbb{N}$ is studied. The exact optimal control is obtained explicitly in a particular case by making use of dynamic programming. An approximation of the random variable T by a sum of i.i.d. exponential random variables is also considered in order to transform the semi-Markov process into a Markovian process.

Keywords*Homing problem, first-passage time, dynamic programming, pure birth process, Weibull distribution.*

Pure and Mixed Nash Equilibria for Dynamic Positional Games on Graphs

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Let $G = (X, E)$ be a directed graph in which an arbitrary vertex $x \in X$ has at least one outgoing directed edge $e = (x, y) \in E$. The vertex set X is divided into m disjoint subsets $X_1, X_2, \ldots, X_m$ ($X = X_1 \cup X_2 \cup \cdots \cup X_m$; $X_i \cap Xj = \emptyset, i \neq j$) which correspond to position sets of m players. On edge set m functions $r^i : F \to R$, $i = 1, 2, \ldots, m$ are defined that assign to each directed edge $e = (x, y) \in E$ the values $r^1_e, r^2_e, \ldots, r^m_e$ that represent the rewards for the corresponding players $1, 2, \ldots, m$ when a move through a directed edge $e = (x, y) \in E$ from x to y is made. On G the following m-person dynamic game is considered: The game starts at a given position $x_0 \in X$ at the moment of time $t = 0$, where the player $i \in \{1, 2, \ldots, m\}$ who is the owner of position x_0 makes a move from x_0 to a neighbor position $x_1 \in V$ through a directed edge $e_0 = (x_0, x_1) \in E$ and players $1, 2, \ldots, m$ receive the corresponding rewards $r^1_{e_0}, r^2_{e_0}, \ldots, c^m_{r_0}$. Then at the moment of time $t = 1$ the player $k \in \{1, 2, \ldots, m\}$ who is the owner of position x_1 makes a move from x_1 to $x_2 \in X$ through the directed edge $e_1 = (x_1, x_2) \in E$ and players $1, 2, \ldots, m$ receive the corresponding rewards $r^1_{e_1}, r^2_{e_1}, \ldots, r^m_{e_1}$, and so on indefinitely. Such a play of the game on G produces the positions $x_0, x_1, x_2, \ldots, x_t \ldots$ induced by strategies of moves of players $1, 2, \ldots, m$. If in this game each player $i \in \{1, 2, \ldots, m\}$ makes moves from his positions set in order to maximize $\omega^i_{x_o} = \lim_{t\to\infty} \inf \frac{1}{t} \sum_{\tau=0}^{t-1} r^i_{e_\tau}$, $i = 1, 2, \ldots, m$, then such a game on G we call *mean positional game*[1,2]). If each player $i \in \{1, 2, \ldots, m\}$ makes moves from his positions set in order to maximize $\psi^i_{x_o} = \sum_{\tau=0}^{\infty} \lambda^i r^i_{e_\tau}$, $i = 1, 2, \ldots, m$, with given discount factor λ, $0 < \lambda < 1$, then such a game on G we call *discounted positional game*[2,3]). We derived conditions for the existence of Nash equilibria in pure and mixed stationary strategies for such games [3].

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Multi-agent system for modeling and decision support within a university admission committee

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This paper presents the development of a multi-agent system for modelling and supporting the decision-making process within a university admission committee. The research is motivated by the need to efficiently manage educational data flows, admission applications, candidates' preferences, institutional capacities, and administrative constraints in an environment characterized by dynamic conditions, uncertainty, and interdependent decision-making. The proposed system integrates specialized autonomous agents representing candidates, study programmes, the admission committee, institutional policies, and available resources. Each agent possesses its own perception, analysis, and decision-making mechanisms, while the overall behaviour of the system emerges through cooperation, information exchange, coordination, and distributed optimisation. Building upon the proposed model, the multi-agent architecture can be further enhanced with machine learning techniques and neural networks to estimate admission demand, predict the evolution of student cohorts, and support the optimal allocation of candidates to study programmes. The proposed solution contributes to improving the transparency, adaptability, and efficiency of the admission process by providing formal and intelligent decision support for university management.
Keywords*multi-agent system, university admission, admission committee, decision support, educational modelling, autonomous agents, distributed optimisation, machine learning, neural networks, educational management.*

Robust PM Concentration Prediction under Missing Data Conditions

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Accurate prediction of PM2.5 concentrations is essential for air quality assessment, environmental management, and public health protection. However, air quality monitoring datasets frequently contain missing measurements caused by sensor failures, communication interruptions,

calibration procedures, or maintenance activities. Traditional time-series forecasting approaches, such as ARIMA and SARIMA, generally require complete and regularly sampled observations and often experience significant performance degradation when confronted with incomplete datasets. Furthermore, their ability to capture the complex nonlinear relationships among meteorological variables and pollutant concentrations is limited, leading to unsatisfactory predictive performance in real-world applications. This study proposes an XGBoost-based framework for PM2.5 concentration prediction under missing data conditions. The proposed approach exploits the robustness of gradient boosting decision trees to handle incomplete observations while effectively modeling nonlinear interactions among air quality and meteorological factors. A comprehensive experimental evaluation was conducted using air quality monitoring data characterized by different proportions of missing values. The predictive performance of the XGBoost model was compared with that of conventional statistical methods, including ARIMA, SARIMA, VARIMA models.
The obtained results demonstrate that the proposed method consistently outperforms traditional forecasting techniques across all missing-data scenarios. While ARIMA- family-based models exhibit reduced accuracy and limited applicability when data gaps are present, the XGBoost model maintains stable predictive capability and achieves lower forecasting errors. The findings highlight the suitability of machine learning approaches for air quality forecasting in operational environments where missing measurements are unavoidable. These results suggest that XGBoost represents a reliable and effective alternative to conventional statistical models for PM prediction, offering improved robustness, flexibility, and forecasting accuracy in the presence of incomplete monitoring data.
Keywords *PM prediction, air quality forecasting, missing data, XGBoost, machine learning, ARIMA, SARIMA, VARIMA, environmental monitoring.*

Regression Methods for Processing Solar Spectral Data

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Based on solar spectral data and using a quantum spectral model [1,2] and several spectroscopic and statistical assumptions [2-4], we estimate the parameters of the regression curves for multiples in the Lyman to Humphreys spectral series. We also present an algorithm for quantum spectral reconstruction of multiplets in fine structure approximation for any spectral series Sn of the H-I spectrum, n being the order of the spectral series. To provide a global statistical picture of the first six series of the H-I spectrum, a graphical representation is used. These results confirm the measurements of solar data from the ACE mission.

Keywords*Statistics-regression, spectral model, Humphreys series, solar data, algorithm*

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Reliability of dual Serial – Parallel and Parallel – Serial networks with Weibull distributed units lifetimes and their statistical analysis

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This paper introduces two new dynamic reliability models for dual parallel–series and series–parallel networks, in which the lifetimes of the network units follow a Weibull distribution. Analytical expressions for the cumulative distribution functions, reliability functions, and hazard rates of the network lifetimes are derived. Maximum likelihood estimators for the unknown Weibull parameters are obtained for both complete and censored lifetime data. The theoretical developments are complemented with numerical examples that illustrate the computation and performance of the proposed estimators.

Keywords *Weibull distribution, serial – parallel networks, parallel – serial networks, reliability, maximum likelihood estimator.*

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Optimal Ordering of Parameters in Spider Charts for Confidence-Aware Clinical Stratification

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Multidimensional clinical states are commonly represented by scalar scores obtained by aggregating multiple diagnostic parameters. Although convenient for classification, such aggregation inevitably reduces the amount of information preserved from the original multidimensional description, potentially decreasing confidence in subsequent clinical stratification. Spider charts provide a geometric representation of multidimensional data, but their polygon geometry and enclosed area depend on the ordering of the parameters. Traditionally, this order dependence has been regarded as a limitation.

In this work, we suggest a different perspective. We reinterpret axis-order dependence not as a drawback but as an optimization opportunity for preserving clinically meaningful information

during knowledge representation. The proposed approach generates all non-equivalent cyclic permutations of the diagnostic parameters and evaluates each ordering according to its ability to separate predefined clinical classes. The optimal parameter ordering is the one that minimizes the overlap between the polygon-area intervals associated with different clinical severity groups, thereby maximizing the discriminative capability of the representation.
This method was evaluated using multidimensional Doppler ultrasound data describing portal hemodynamics in patients with liver cirrhosis. The results demonstrate that appropriate parameter ordering substantially improves the separation of clinical severity classes without altering the original diagnostic values. Consequently, spider charts become more than a visualization tool – they function as an optimized geometric representation that preserves multidimensional relationships lost during scalar aggregation.
The proposed approach provides an objective criterion for spider chart design and suggests a new direction for confidence-aware multidimensional stratification, in which the optimization target is not only numerical classification accuracy, but also the preservation of clinically meaningful structure throughout the knowledge formalization.

Keywords*knowledge formalization, confidence-aware multidimensional stratification, spider charts, axis-order dependence.*

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Fuzzy method for solving multi-criteria linear optimization in integers

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The multi-criteria optimization models currently are increasing demand. This is primarily due to the fact that numerous problems of the various practical fields require acceptable solutions from the multiple several criteria [1]. Many and varied practical problems lead to some multi-criteria linear optimization models in integers [2]. In this paper we propose the combinatorial fuzzy method for solving optimization problems of this type. The proposed algorithm leads to the determination of a compromise integer solution, which is located closest to the optimal solution of each criterion, taking into account the best and the worst values of each criterion. Thus, there are more of combination possibilities to find the optimization model considering some objective functions solving in integer, but another in real aria. This is why the final compromise solution may be no single. This fact increase the algorithm importance, especially because a decision maker can obtain more compromise solutions. The theoretical justifications of the proposed method, as well as a solved example applying this method, are presented in the paper.

Keywords *fuzzy programming, fuzzy cost, fractional transportation problem, efficient solution.*

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Audio-to-Haptic Control Streams: A Computational Prototype

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Audio-to-haptic translation concerns the transformation of audio signals into tactile or vibrotactile control representations. Such transformations are important in multimodal interaction, sensory augmentation, assistive technologies, environmental sound awareness, and accessibility-oriented interfaces. However, audio-to-haptic mapping is not a uniquely defined computational problem: different descriptors, mapping rules, actuator types, body locations, and usage contexts may lead to substantially different design decisions.

We present a rule-based computational prototype for generating simulated haptic control streams from audio input. The prototype receives an audio file, extracts frame-level and event-level descriptors, applies robust normalization and smoothing, and maps the resulting features to actuator-agnostic haptic parameters. These parameters include normalized amplitude, carrier frequency, pulse rate, duty cycle, attack and release times, texture mix, onset flags, event identifiers, and virtual channel assignment.

The prototype should be interpreted as a computational demonstrator rather than as a validated haptic rendering system. It does not include hardware actuation, actuator calibration, perceptual evaluation, user testing, or accessibility assessment. Therefore, the results support claims about computational feasibility, reproducibility, transparency, and inspectability of the mapping workflow, but not claims about tactile perceptibility, usability, accessibility benefit, or hardware performance.

The final discussion identifies several directions for further research, including connection to physical actuators, calibration of ERM (Eccentric Rotating Mass), LRA (Linear Resonant Actuator), piezoelectric, and force-feedback devices, comparison of alternative mapping strategies, perceptual studies, and user-centered evaluation in assistive contexts such as alerting, navigation, environmental sound awareness, and multimodal communication.

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A Study of Leader–Antileader Configurations in Comonotone and Antimonotone Samples

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The purpose of the present paper is to investigate leader and antileader properties for a family of bivariate populations obtained by mixing comonotone and antimonotone components. More precisely, we consider random points of the form $\mathbf{Z} = (U, U)\, 1_A + (U, f(U))\, 1_{A^c}$ where U is uniformly

distributed on $[0,1]$, A is an event with probability p and $f : [0,1] \to [0,1]$ is a continuous decreasing function. Thus, with probability p the observation belongs to the diagonal of the unit square, while with probability $1-p$ it belongs to the graph of the decreasing function f. This construction generates a natural interpolation between a purely comonotone and a purely antimonotone population.
For a sample $\mathcal{S}_n = \{\mathbf{Z}_1, ..., \mathbf{Z}_n\}$, we investigate three fundamental probabilities: the probability that $\mathcal{S}_n$ has a leader - denoted by a_n, the probability that it has an antileader - denoed by b_n, and c_n - the probability that both properties occur simultaneously. We derive exact formulas for these quantities and we also study their asymptotic behavior. Finally, we prove that $\lim c_n \geq (\lim a_n)(\lim b_n)$.

The present work differs from the classical theory of records in several respects. First, the observations are two-dimensional and are ordered only partially. Second, the support of the distribution is not a smooth two-dimensional domain but a mixture of two one-dimensional structures corresponding to opposite dependence regimes. Third, the main objects of interest are not record values themselves but the existence of maximal and minimal elements in the sample. Consequently, the problem combines ideas from record theory, geometric probability and dependence modeling.
Acknowledgments. This work was carried out within the framework of the "Statistics, Probability and Operational Research" research program of the Romanian Academy.

6. Algebra, Logic, Geometry (with applications)

On the Hilbert depth of quadratic and cubic functions

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Given a numerical function $h : \mathbb{N} \to \mathbb{N}$ with $h(0) > 0$, the Hilbert depth of h is

$$\operatorname{hdepth}(h) = \max\{d \ : \ \sum_{j=0}^{k}(-1)^{k-j}\binom{d-j}{k-j}h(j) \geq 0 \text{ for all } k \leq d\}.$$

See [1] for further details. In this note, we study the Hilbert depth of the functions

$$h_2 : \mathbb{N} \to \mathbb{N},\ h_2(j) = aj^2 + bj + e,\ j \geq 0,$$
$$h_3 : \mathbb{N} \to \mathbb{N},\ h_3(j) = aj^3 + bj^2 + cj + e,\ j \geq 0,$$

where a, b, c, e are some integers with $a, e > 0$.

Our main results are the following:

Theorem 1. *With the above notations, we have that:*

(1) If $b \geq 0$, *then* $\operatorname{hdepth}(h_2) \leq 8$.

(2) If $b < 0$ *and* $b^2 \leq 4ae$, *then* $\operatorname{hdepth}(h_2) \leq 11$.

(3) If $b < 0$ *and* $b^2 > 4ae$, *then* $\operatorname{hdepth}(h_2) \leq 13$.

Theorem 2. *With the above notations, we have that:*

(1) If $b, c \geq 0$, *then* $\operatorname{hdepth}(h_3) \leq 16$.

(2) If $b < 0$ *and* $b^2 \leq 3ac$, *then* $\operatorname{hdepth}(h_3) \leq 67$.

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On Cartan structural equations of an acm-structure on a totally umbilical hypersurface of an almost Hermitian manifold

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Since the middle of the last century, it has been known that an almost contact metric (acm-) structure is induced on every orientable hypersurface of an almost Hermitian manifold [1]. In her fundamental research [2], L.V. Stepanova obtained the so-called Cartan structural equations for the almost contact metric structure induced on a hypersurface of an arbitrary almost Hermitian manifold. The very complicated derivation of these structural equations was only slightly improved in [1]. The main result that we will present is a specification of the above-mentioned structural equations for the important case when an acm-hypersurface of an almost Hermitian manifold is its totally umbilical submanifold. These equations immediately allow us to obtain a lot of generalizations of results obtained for acm-hypersurfaces of specific almost Hermitian manifolds (e.g., [3] and [4]). On the other hand, another natural continuation of our work could be the derivation of Cartan structural equations for acm-hypersurfaces of almost Hermitian manifolds for every of the sixteen Gray-Hervella classes [5].

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Subcategories $\mathcal{B}$-semireflexive

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The paper investigates the $\mathcal{B}$-semireflexive subcategories in the category $\mathcal{C}_2\mathcal{V}$ of locally convex topological vector spaces, with respect to a given class of bimorphisms [1,2]. The study begins

with the known characterizations of semireflexive spaces and extends them in the categorical context by introducing the notions of semireflexive product and its dual. Reflective and coreflective subcategories are examined through the lens of morphism classes $\varepsilon\mathcal{R}$ and $\mu\mathcal{K}$, establishing a bijective correspondence between certain reflective and coreflective structures. Several theorems are presented that characterize the situations when a subcategory is $(\varepsilon\mathcal{L})$-semireflexive, along with examples including locally complete spaces, inductively semireflexive spaces, $\mathcal{B}$-inductively semireflexive spaces, and p-semireflexive spaces. The results provide categorical descriptions and structural properties of these subcategories, highlighting their interrelations and the role of semireflexive products in their classification.

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On the functorial isomorphism between the lattices of $\mathcal{P}$-quotients and of saturated sets of morphisms.

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Let $\mathcal{C}$ be a small, locally and colocally category with products, $(\mathcal{P},\mathcal{I})$ a factorization structure in category $\mathcal{C}$, $\mathcal{R}$ a $\mathcal{P}$-reflective subcategory and B - an $\mathcal{I}$-cogenerator of the subcategory $\mathcal{R}$. For every object X we study two ordered structures: the class $\mathbb{P}(X)$ of $\mathcal{P}$-morphisms $p : X \to Y$ with $Y \in |\mathcal{R}|$, and the class $\mathcal{S}(X)$ of saturated subsets of $\mathcal{C}(X, B)$. We prove that, under the certain conditions PS (for any morphism $p : X \to Z \in \mathcal{P}$ and any morphism $f : Y \to Z \in \mathcal{C}$ there exists the pushout square built on these morphisms) or PD (for any morphism $p : Z \to X \in \mathcal{P}$ and any morphism $f : Z \to Y \in \mathcal{C}$ there exists the pullback square built on these morphisms), both are complete lattices and that the assignments $X \to \mathbb{P}(X)$ and $X \to \mathcal{S}(X)$ extend to covariant functors $\mathcal{F}$, $\mathcal{S} : \mathcal{C} \to \mathcal{L}at$, as well as to contravariant functors $\mathcal{F}^c$, $\mathcal{S}^c : \mathcal{C} \to \mathcal{L}at$. The main results establish that the functorial morphisms $\alpha : \mathcal{S} \to \mathcal{F}$, $\beta : \mathcal{F} \to \mathcal{S}$ and their contravariant analogues $\overline{\alpha} : \mathcal{S}^c \to \mathcal{F}^c$ and $\overline{\beta} : \mathcal{F}^c \to \mathcal{S}^c$ are mutually inverse, yielding functorial isomorphisms $\mathcal{F} \cong \mathcal{S}$ and $\mathcal{F}^c \cong \mathcal{S}^c$. Thus, the reflective quotients admit two equivalent descriptions: an external one, by morphisms of the class $\mathbb{P}$, and an internal one, by saturated families of morphisms into the cogenerator B. Examples in the categories of sets, zero-dimensional spaces and Tikhonov spaces illustrate the results.

Keywords *factorization structure, reflective subcategory, cogenerator, complete lattice, functorial isomorphism, saturated set.*

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Note on algebraic and topological quasigroups obeying certain laws

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A non-empty set G is said to be a *groupoid* with respect to a binary operation denoted by $\{\cdot\}$, if for every ordered pair (a, b) of elements of G there is a unique element $ab \in G$.

A quasigroup is a binary algebraic structure in which one-sided multiplication is a bijection in that all equations of the form $ax = b$ and $ya = b$ have unique solutions [1].

A groupoid G is called a primitive groupoid with divisions, if there exist two binary operation $l : G \times G \to G$, $r : G \times G \to G$ such that $l(a, b) \cdot a = b, a \cdot r(a, b) = b$ for all $a, b \in G$. Thus a primitive groupoid with divisions is a universal algebra with three binary operations.

A primitive groupoid G with divisions is called a quasigroup if the equations $ax = b$ and $ya = b$ have unique solutions. In a quasigroup G the divisions l, r are unique. If the multiplication operation in a quasigroup $(G, \cdot)$ with a topology is continuous, then G is called a semitopoligical quasigroup. If in a semitopological quasigroup G the divisions l and r are continuous, then G is called a topological quasigroup.

A groupoid $(G, \cdot)$ is called:
-*medial* if $xy \cdot zt = xz \cdot yt$ for all $x, y, z, t \in G$.
-*paramedial* if $xy \cdot zt = ty \cdot zx$ for all $x, y, z, t \in G$.
-*bicommutative* if $xy \cdot zt = tz \cdot yx$ for all $x, y, z, t \in G$.
-*left semimedial* if $xx \cdot yz = xy \cdot xz$ for all $x, y, z \in G$.
-*Schweitzer* if $yx \cdot yz = zx$ for all $y, x, z \in G$.
-*left Bol* if $x(y(xz)) = (x(yx))z$ for all $x, y, z \in G$.
-*AD-groupoid* if $a \cdot bc = c \cdot ba$ for all $a, b, c \in G$.
-*AG-groupoid* if $ab \cdot c = cb \cdot a$ for all $a, b, c \in G$.

The concept of (n, m)-identities was introduced by M.M. Choban and L.L. Chiriac in [2].

We are interested to study which kind of weakened associativity ensures us to obtain structural theorems for algebraic and topological quasigroups which are similar to proprieties of algebraic and topological groups.

Quasigroup $(G, \cdot)$ is a $T - quasigroup$ if and only if there exist an abelian group $(G, +)$, its automorphisms φ and ψ, and a fixed element $a \in G$ such that $x \cdot y = \varphi(x) + \psi(y) + a$ for all $x, y \in G$.

We research $T-quasigroups$ with AD identity (or Abel-Grassmann's right invertive groupoid identity)and Schweitzer identity.

Theorem 1. *Let $(G,\cdot)$ be a $T-quasigroup$. Then $(G,\cdot)$ is $AD-quasigroup$ if and only if for any of its $T-forms$ $(G,+,\varphi,\psi)$, $\varphi(x)=\psi(x)^2$.*

Theorem 2. *Let $(G,\cdot)$ be a $T-quasigroup$. Then $(G,\cdot)$ is Schweitzer quasigroup if and only if for any of its $T-forms$ $(G,+,\varphi,\psi)$, $\varphi^2(y)+\psi\varphi(y)=0$, $\varphi(\psi(x))=\psi(x)$, $\psi^2(z)=\varphi(z)$.*

The Theorems 1 and 2 were proven in [6].

We have used the concept of a special direct product of an Abelian topological group G and proved that a binary operation can be defined on the set $G\times G$, such that the new algebraic structure is a non-associative topological quasigroups obeying certain laws.

Theorem 3. *Let $(G,+,\tau)$ be a commutative topological group where G is not a singleton. For (x_1,y_1) and (x_2,y_2) in $G\times G$ define*

$$(x_1,y_1)\circ(x_2,y_2)=(x_1+x_2,y_1-y_2).$$

Then $(G\times G,\circ,\tau_G)$, relative to the product topology τ_G, is a medial, paramedial, bicommutativ, left Bol, left-semimedial and AD non-associative topological quasigroup. Moreover, if (G,τ) is $T_i-space$, then $(G\times G,\tau_G)$ is $T_i-space$ too, where $i=1,2,3,3.5$.

Next, we study under which conditions the binary groupoid G with a locally compact Hausdorff topology can be "transformed" into a *Schweitzer* topological quasigroup?

The maps $r_a:G\to G$, $x\mapsto xa$ and $l_a:G\to G$, $x\mapsto ax$ are called the right translation and left translation by a, respectively.

Theorem 4. *Let τ be a locally compact topology defined on a multiplicative groupoid $(G,\cdot)$ with right identity $e\in G$ and the following conditions hold:*

1. $x^2=e$ for every $x\in G$,

2. $(yz)\cdot(yx)=xz$ for all $x,y,z\in G$,

3. if $xa=ya$ then $x=y$ for all $x,y,a\in G$,

then $(G,\cdot,\tau)$ is a topological Schweitzer quasigroups with a $(2,1)$-identity e if and only if r_a is open and continous for each $a\in G$.

The results established here are related to the work in ([3,4,5,6]).

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Several classes of special Hermitian metrics

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Special Hermitian metrics provide a flexible framework for the study of compact complex manifolds that carry no Kähler structure. Weakening the Kähler condition $d\omega = 0$ by imposing weaker differential conditions on the fundamental form leads to several well-studied classes of Hermitian metrics, including locally conformally Kähler, Gauduchon and k-Gauduchon, pluriclosed (or strong Kähler with torsion), astheno-Kähler, balanced, and locally conformally balanced metrics. In the present note, we recall the definitions of these classes and describe their mutual relations — for instance, pluriclosed and astheno-Kähler metrics are, respectively, the 1- and $(n-2)$-Gauduchon metrics, while a Hermitian metric which is at once balanced and pluriclosed, or balanced and astheno-Kähler, is forced to be Kähler. We illustrate each class by representative examples on Hopf, Inoue, Calabi–Eckmann, twistor and nilmanifold spaces. We then concentrate on the class of *EP-like* metrics, whose fundamental form ω satisfies $\partial\overline{\partial}\omega^k = 0$ for every $1 \leq k \leq n-1$. An EP-like metric is simultaneously pluriclosed and astheno-Kähler. We describe their explicit construction on Endo–Pajitnov manifolds, and their stability under small deformations of the complex structure and under blow-ups along complex submanifolds.

Keywords *Hermitian metric, pluriclosed, SKT, astheno-Kähler, balanced, Gauduchon, EP-like metric, Endo–Pajitnov manifold*

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Construction of the Riemann Map

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This presentation introduces a constructive method for approximating the Riemann map and discusses its potential applications. Rather than proving the classical Riemann Mapping Theorem for an arbitrary simply connected domain in $\mathbb{C}$, we construct a sequence of increasingly accurate approximations to the Riemann map. The method begins by approximating the domain with a polygonal region obtained from a Whitney decomposition. The Schwarz–Christoffel formula is then applied to compute the conformal map onto the polygonal approximation, and the Carathéodory Convergence Theorem is used to obtain the Riemann map as the limit of the resulting sequence of conformal maps. The proposed approach provides a constructive framework for approximating conformal maps and has promising computational applications.
For further details, see P. V. Dovbush and S. G. Krantz, *Construction of the Riemann Map*, Complex Analysis and Operator Theory **18** (2024), Article 165.

Prime numbers sequence approximation

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Let $\mathcal{P}$ be the set of prime numbers. The Chebyshev polynomials of first and second kinds $T_n(x)$ and $U_n(x)$ play an important role in mathematics, especially in trigonometry ([2],[3]) analysis and ordinary differential equations ([1]). The Chebyshev polynomials of second kind are also utilized in numerical analysis, quantum mechanics with an empahasis to the quantum theory of angular momentum ([4]), in engineering and electromagnetism, more specific in numerical methods in electromagnetic theory, fluid flow simulations ([5]) and in signal processing specificallly in approximation theory to minimize approximation errors ([6]).

In this note we express prime numbers in terms of certain sequences more exactly we prove that for any prime number $p \in \mathcal{P}$ there exists a sequence of rationals $(x_n)_{n\in\mathbb{N}}$ with $x_0 = x_1 = 1$ such that $p = x_n \cdot x_{n+2} + x_{n+1}^2$. We prove that moreover $(x_n)_{n\in\mathbb{N}}$ are in fact integers and we derive a formula for the general term using certain linear reccurences, proving the combinatorial formula : $x_n = (-1)^{\lfloor \frac{n-1}{2} \rfloor} \cdot \sum_{k=0}^{n-1} (-1)^k \cdot \binom{2n-2-k}{k} \cdot p^{n-1-k}$. Next we study their properties proving that the sum of the coefficients of any term of $(x_n)_{n\in\mathbb{N}}$ lives in the set $\{-1, 0, 1\}$ also obtaining that this sum is:

$$\begin{cases} 1 & n \equiv 1(3) \\ 0 & n \equiv 2(3) \\ -1 & n \equiv 0(3). \end{cases}$$

After setting $P_n := \epsilon_n \cdot x_n$ where $\epsilon_n := (-1)^{\lfloor \frac{n-1}{2} \rfloor}$, $n \geq 1$, we prove the recurrence $P_{n+1} = (p-2) \cdot P_n - P_{n-1}$ and furthermore we relate the polynomials $(P_n)_{n\in\mathbb{N}^*}$ with Chebyshev polynomials of the second kind proving that $P_n(p) = U_{n-1}(\frac{p-2}{2}) + U_{n-2}(\frac{p-2}{2})$.

Finally, if g is the gap between two consecutive primes p and $p + g$, by setting the difference $d_n := P_n(p + g) - P_n(p)$, we prove the nonhomogeneous recurrence:

$$\begin{cases} d_0 = 0 \\ d_1 = 0 \\ d_2 = g \\ d_{n+2} = (p-2) \cdot d_{n+1} - d_n + g \cdot P_{n+1}(p + g). \end{cases}$$

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Keywords*prime number, sequence, Chebyshev polynomial, linear reccurence*

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Autotopies and automorphisms of the loop of pairs of some DK-ternar

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All necessary definitions can be found in the works [3, 2].

Definition 1. *A set $\langle P, L, I\rangle$ of objects of 2 types (P - points and L - lines) with an incidence relation I is called a* ***net*** *(**algebraic net**) if the following conditions are satisfied:*

1. *The set L can be separated into disjoint classes $L_1, L_2, \ldots, L_k$.*
2. *Two lines from different classes are incident to exactly one point.*
3. *Each point $X \in P$ is incident to exactly one line from each class L_i .*

The number k is called the *genus* of the net N, and the net N is then called a k-net. It is known [2] that each line L contains the same number n of points, and the number of lines in each class L_i is also equal to n . The number n is called the *order* of the net.

The inequality $k \leqslant n+1$ always holds for k-nets. If $k = n+1$, then such k-net is called an *affine plane*. Let us remember the basic facts from the theory of the finite affine planes and its coordinatization (see [6]).

Definition 2. *[6] A system $\langle E, (x,t,y), 0, 1\rangle$ is called a* ***DK-ternar*** *(i.e. a set E with ternary operation (x,t,y) and distinguished elements $0, 1 \in E$), if the following conditions hold:*

1. $(x, 0, y) = x$,
2. $(x, 1, y) = y$,
3. $(x, t, x) = x$,
4. $(0, t, 1) = t$,

5. *if a, b, c, d are arbitrary elements from E and $a \neq b$, then the system*

$$\begin{cases} (x, a, y) = c \\ (x, b, y) = d \end{cases}$$

has an unique solution in $E \times E$.

Definition 3. *[?] A set M of permutations on a set X is called **sharply 2-transitive** if for any two pairs (a, b) and (c, d) of different elements from X there exists an unique permutation $\alpha \in M$ satisfying the following conditions:*

$$\alpha(a) = c, \qquad \alpha(b) = d.$$

Lemma 2. *[6] Let π be an arbitrary finite affine plane. We can introduce on the plane π the coordinates $(a, b), (m), (\infty)$ for points and $[a, b], [m], [\infty]$ for lines (where the set E is a finite set with the distinguished elements $0, 1$ and $a, b, m \in E$) such that if we define a ternary operation (x, t, y) on the set E by the formula*

$$(x, t, y) = z \quad \overset{def}{\Longleftrightarrow} \quad (x, y) \in [t, z],$$

then the system $\langle E, (x, t, y), 0, 1\rangle$ be a DK-ternar.

Now let a system $\langle E, (x, t, y), 0, 1\rangle$ be a DK-ternar. Let us define the following binary operation (x, ∞, y) on the set E:

$$\begin{gathered} (x, \infty, 0) \overset{def}{=} x, \\ \begin{cases} (x, \infty, y) = u \\ (x, y) \neq (u, 0) \end{cases} \overset{def}{\Longleftrightarrow} \begin{gathered} (x, t, y) \neq (u, t, 0) \\ \forall t \in E. \end{gathered} \end{gathered}$$

Lemma 3. *[6] Operation (x, ∞, y) satisfies the following conditions:*

1. $\begin{cases} (x, \infty, y) = (u, \infty, v) \\ (x, y) \neq (u, v) \end{cases} \Longleftrightarrow \begin{gathered} (x, t, y) \neq (u, t, v) \\ \forall t \in E. \end{gathered}$
2. $(x, \infty, x) = 0$.
3. *if a, b, c are arbitrary elements from E, then the system*

$$\begin{cases} (x, a, y) = b \\ (x, \infty, y) = c \end{cases}$$

has a unique solution in $E \times E$.

Let $\langle E, (x, t, y), 0, 1\rangle$ be a finite DK-ternar. Let us introduce points (a, b) and lines $[a, b], [m]$ (where $a, b, m \in E$) and define the following incidence relation I between points and lines:

$$\begin{gathered} (a, b)\ I\ [c, d] \quad \Longleftrightarrow \quad (a, c, b) = d, \\ (a, b)\ I\ [d] \quad \Longleftrightarrow \quad (a, \infty, b) = d, \end{gathered} \tag{1}$$

Lemma 4. *[6] The incidence system $\langle X, L, I\rangle$, where*

$$\begin{gathered} X = \{(a, b) \mid a, b \in E\}, \\ L = \{[a, b], [m] \mid a, b, m \in E\}, \\ I \text{ is the incidence relation, defined above in (1),} \end{gathered}$$

is an affine plane.

Lemma 5. *[6] (**Cell permutations**) Let the system $\langle E,(x,t,y),0,1\rangle$ be a finite DK-ternar. Let a,b be an arbitrary elements from E and $a\neq b$. Then every unary operation $\alpha_{a,b}(t)=(a,t,b)$ is a permutation on the set E.*

Lemma 6. *[6] Cell permutations $\{\alpha_{a,b}\}_{a,b\in E,\ a\neq b}$ of the finite DK-ternar $\langle E,(x,t,y),0,1\rangle$ satisfy the following conditions:*

1. *All cell permutations are distinct;*
2. *The set M of all cell permutations is sharply 2-transitive on the set E;*
3. *A permutation $\alpha_{a,b}$ is a fixed-point-free cell permutation on the set E iff the following condition holds*
$$(a,\infty,b)=(0,\infty,1).$$
4. *There exists the fixed-point-free permutation ν_0 on the set E such that we can represent the set A of all fixed-point-free cell permutations together with the identity cell permutation $\alpha_{0,1}$ in the following form:*
$$A=\{\alpha_{a,b}\,|\,b=\nu_0(a),\quad a\in E\}=\{\alpha_{a,\nu_0(a)}\}_{a\in E}.$$

Lemma 7. *[6] Let $M=\{\alpha_{a,b}\}_{a,b\in E,\ a\neq b}$ be a set of permutations on the set E (E is a finite set with distinguished elements 0 and 1), and the following conditions hold:*

1. *$\alpha_{0,1}=id$.*
2. *$\alpha_{a,b}(0)=a,\ \alpha_{a,b}(1)=b$.*
3. *The set M is a sharply 2-transitive set of permutations on E.*

   Let us suppose by definition:
$$(x,t,y)\overset{def}{=}\alpha_{x,y}(t)\quad if\quad x\neq y,$$
$$(x,t,x)\overset{def}{=}x.$$

   Then the system $\langle E,(x,t,y),0,1\rangle$ is a finite DK-ternar.

Next theorem show a connection between a finite sharply 2-transitive sets of permutations and loop transversals in the symmetric group S_n.

Theorem 1. *[5] Let E be a finite set and $\operatorname{card} M=n$. Then the following conditions are equivalent:*

1. *A set T of permutations of degree n is a sharply 2-transitive set of permutations on the set E and $id\in T$.*
2. *A set T of permutations of degree n is a loop transversal in S_n to $St_{a,b}(S_n)$ (where a,b are arbitrary fixed elements from E and $a\neq b$).*
3. *A system $\langle E\times E-\{\triangle\},\overset{(T)}{\cdot},\langle a,b\rangle\rangle$ is a sharply 2-transitive permutation loop of degree n (a definition of permutation loop see in [**?**, **?**, **?**]).*

Lemma 8. *[6] Let $T_{a,b} = \{\alpha_{x,y}\}_{x,y\in E,\ x\neq y}$ be a loop transversal in S_n to $St_{a,b}(S_n)$ (where a, b are arbitrary fixed elements from E and $a \neq b$). Let a system $\langle E \times E - \{\triangle\}, \overset{(T_{a,b})}{\cdot}, \langle a, b\rangle\rangle$ be a loop transversal operation corresponding to the transversal $T_{a,b}$. Then*

$$\langle x, y\rangle \overset{(T_{a,b})}{\cdot} \langle u, v\rangle = \langle \alpha_{x,y}(u), \alpha_{x,y}(v)\rangle. \tag{2}$$

As it is shown above, there exist a 1-1 correspondences between

- a finite projective plane π of order n;
- a finite DK-ternar $\langle E, (x, t, y), 0, 1\rangle$ which gives a coordinatization of the projective plane π;
- a sharply 2-transitive permutation loop $L = \{\alpha_{a,b}\}_{a,b\in E,\ a\neq b}$ of cell permutations of the DK-ternar $\langle E, (x, t, y), 0, 1\rangle$;
- a loop transversal $T_{a,b} = \{\alpha_{x,y}\}_{x,y\in E,\ x\neq y}$ in the symmetric group S_n to $St_{a,b}(S_n)$ (where a, b are arbitrary fixed elements from E and $a \neq b$);
- a loop transversal operation $\langle E \times E - \{\triangle\}, \overset{(T_{a,b})}{\cdot}, \langle a, b\rangle\rangle$ corresponding to the transversal $T_{a,b}$ (in [6] this loop is called a **loop of pairs** of the DK-ternar $\langle E, (x, t, y), 0, 1\rangle$).

Below for simplicity we shall consider that $\langle a, b\rangle = \langle 0, 1\rangle$.

Lemma 9. *A set*

$$H_0^* = \{\langle 0, a\rangle \mid a \in E \setminus \{0\}\}$$

forms a subloop in the loop of pairs $L^* = \langle E \times E - \{\triangle\}, \overset{(T_{0,1})}{\cdot}, \langle a, b\rangle\rangle$.

Let us study a set $A = \{\alpha_{a,\nu(a)}\}_{a\in E} \subset L$ of all fixed-point-free permutations and the identity permutation (see Lemma 7).

Lemma 10. *A set $A = \{\alpha_{a,\nu(a)}\}_{a\in E}$ is a loop transversal in the loop $L = \{\alpha_{a,b}\}_{a,b\in E,\ a\neq b}$ to its proper subloop H_0.*

Two following theorems are the main results of this work.

Theorem 2. *Any left translation*

$$L_{(a,\nu(a))}((x, y)) = (a, \nu(a)) \overset{(T_{a,b})}{\cdot} (x, y)$$

(where $a \in E$) generate an autotopy of the loop of pairs of the DK-ternar $\langle E, (x, t, y), 0, 1\rangle$.

Theorem 3. *Any "conjugation"*

$$F_{(0,u)}((x, y)) = (0, u) \overset{(T_{a,b})}{\setminus} ((x, y) \overset{(T_{a,b})}{\cdot} (0, u))$$

(where $u \in E\setminus\{0\}$) generate an automorphism of the loop of pairs of the DK-ternar $\langle E, (x, t, y), 0, 1\rangle$.

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Zero Location for analytic and harmonic polynomial and rational quadrinomials

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Inclusion sectors for the zeros of analytic and harmonic polynomial and rational quadrinomial functions are derived. In addition, we also briefly discuss the number of zeros of such functions, which is, in general, an open problem that, so far,has only been solved for narrowly defined special cases.

On some ideals of the ring of continuous endomorphisms of an LCA group

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Let $\mathcal{L}$ be the class of locally compact abelian groups. For $X \in \mathcal{L}$, let $c(X)$, $d(X)$, $k(X)$, $m(X)$, and $E(X)$ denote respectively the connected component of zero in X, the maximal divisible subgroup of X, the subgroup of compact elements of X, the smallest closed subgroup K of X such that the quotient group X/K is torsion free, and the ring of continuous endomorphisms of X. If

Y is another group in $\mathcal{L}$, we let $H(X,Y)$ denote the group of continuous homomorphisms from X to Y. Consider the following ideals:

$$A = \{f \in E(X) \mid k(X) \subseteq \ker(f)\},$$
$$B = \{f \in E(X) \mid \mathrm{im}(f) \subseteq c(X)\},$$
$$C = \{f \in E(X) \mid \mathrm{im}(f) \subseteq k(X)\},$$
$$D = \{f \in E(X) \mid c(X) \subseteq \ker(f).$$

Theorem 1. *For a non-zero group $X \in \mathcal{L}$, the following conditions are equivalent:*

(i) $A = \{0\}$.

(ii) Either $k(X) = X$, *or* $\{0\} \subsetneq k(X) \subsetneq X$ *and* $H(X/k(X), X) = \{0\}$.

Corollary 1. *Let $X \in \mathcal{L}$ be such that $k(X) \neq \{0\}$. If $A = \{0\}$, then $d(X) = \{0\}$.*

Theorem 2. *For a non-zero group $X \in \mathcal{L}$, the following conditions are equivalent:*

(i) $B = \{0\}$.

(ii) Either $c(X) = \{0\}$, *or* $\{0\} \subsetneq c(X) \subsetneq X$ *and* $H(X, c(X)) = \{0\}$.

Corollary 2. *Let $X \in \mathcal{L}$ be such that $c(X) \neq X$. If $B = \{0\}$, then $m(X) = X$.*

Theorem 3. *For a non-zero group $X \in \mathcal{L}$, the following conditions are equivalent:*

(i) $C = \{0\}$.

(ii) Either $k(X) = \{0\}$, *or X is topologically isomorphic with a group of the form* $\mathbb{R}^n \times \mathbb{Q}^{(\mu)} \times Y$, *where* $n \in \mathbb{N}$, *μ is a cardinal number, and Y is a group in $\mathcal{L}$ such that* $\{0\} \subsetneq k(Y) \subsetneq Y$, $d(Y) \subseteq k(Y)$, *and* $H(Y, k(Y)) = \{0\}$.

Corollary 3. *Let $X \in \mathcal{L}$ be such that $k(X) \neq X$. If $A(X) = \{0\}$, then $d(X) = \{0\}$.*

Theorem 4. *For a non-zero group $X \in \mathcal{L}$, the following conditions are equivalent:*

(i) $D = \{0\}$.

(ii) Either $c(X) = X$, *or X is topologically isomorphic with a group of the form* $\mathbb{R}^n \times \mathbb{Q}^{(\mu)} \times Y$, *where* $n \in \mathbb{N}$, *μ is a cardinal number, and Y is a group in $\mathcal{L}$ such that* $\{0\} \subsetneq k(Y) \subsetneq Y$, $d(Y) \subseteq k(Y)$, *and* $H(Y, k(Y)) = \{0\}$.

Corollary 4. *Let $X \in \mathcal{L}$ be such that $c(X) \neq X$. If $D = \{0\}$, then $m(X) = \{0\}$.*

On 4-quasigroups with exactly twenty-four distinct parastrophes

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C.C. Lindner and D. Steedley [1] showed that the possible number of distinct parastrophes of a binary quasigroup is 1,2, 3 or 6 (i.e. divides 3!) and completely characterized the spectrum of finite binary quasigroups with a prescribed number of distinct parastrophes. In the same paper, the problem of generalizing the two aspects to the n-ary case is raised.
M. McLeish proved that the maximum possible number of distinct parastrophes of an n-quasigroup divides $(n+1)!$ and investigated the spectrum of finite ternary quasigroups with a prescribed number of distinct parastrophes [2].
4-Ary quasigroups with a prescribed maximum number of distinct parastrophes and their spectrum are studied in the present work. In particular, we prove that there does not exist 4-ary T-quasigroups (Q, A) having exactly: a. s distinct parastrophes, where $s = 2, 3, 4, 6, 8, 12, 15$; b. 10 distinct parastrophes, where $\{\sigma \in S_5 | {}^{\sigma}A = A\} \cong A_4$; c. 20 distinct parastrophes, where $\{\sigma \in S_5 | {}^{\sigma}A = A\} \cong Z_6$; d. 24 distinct parastrophes, where $\{\sigma \in S_5 | {}^{\sigma}A = A\} \cong Z_5$.
Characterizations of 4-ary T-quasigroups with exactly 5, 10 or exactly 20 distinct parastrophes are considered in [3-5]. Necessary and sufficient conditions when a 4-ary T-quasigroup has exactly 24 distinct parastrophes and some estimations of their spectrum are given in the present work:

Theorem 1. *4-ary T-quasigroup (Q, A) with the T-group $(Q, +)$, has exactly 24 distinct parastrophes if and only if there exist $\alpha \in Aut(Q, +)$ and an element $c \in Q$, such that $\alpha \neq I, \alpha^4 \neq \varepsilon, \alpha^5 = I$, where $I(x) = -x, \forall x \in Q$, and its T-forms is one of the following:*
$T_1 = ((Q, +), \alpha, I\alpha^2, \alpha^3, \alpha^{-1}, c)$, $T_2 = ((Q, +), I\alpha^2, \alpha^3, \alpha^{-1}, \alpha, c)$,
$T_3 = ((Q, +), \alpha, I\alpha^2, \alpha^{-1}, \alpha^3, c)$, $T_4 = ((Q, +), I\alpha^2, \alpha^3, \alpha, \alpha^{-1}, c)$,
$T_5 = ((Q, +), \alpha^3, \alpha^{-1}, \alpha, I\alpha^2, c)$, $T_6 = ((Q, +), \alpha^3, \alpha^{-1}, I\alpha^2, \alpha, c)$,

Corollary 1. *There exist 4-ary T-quasigroups with exactly 24 distinct parastrophes of any prime order q, where $q \equiv 1 \pmod 5$.*

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On Some Integration Techniques

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In this paper we combine power series expansion techniques with integral transformations and use the inversion of the order of integration to calculate some interesting integrals.

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On two exponential diophantine equations

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Many mathematicians investigated the non-negative solutions (x, y, z) of Diophantine equations of the form $a^x + b^y = z^2$, where a, b are fixed natural numbers. In 2021, Viriyapong N. and Viriyapong C. proved that the Diophantine equation $n^x + 13^y = z^2$, where $n \equiv 2 \pmod{39}$ and $n+1$ is not a perfect square, has the unique solution $(n, x, y, z) = (2, 3, 0, 3)$. In 2022, Viriyapong N. and Viriyapong C. showed that the Diophantine equation $n^x + 19^y = z^2$, where $n \equiv 2 \pmod{57}$ has the unique solution $(n, x, y, z) = (2, 3, 0, 3)$. In 2023, Suton T. studied the Diophantine equation $n^x + 10^y = z^2$, where $n \equiv 2 \pmod{30}$.

In this paper, we study the non-negative integer solutions of the Diophantine equations $a^x + 22^y = z^2$, where $a \equiv 2 \pmod{66}$ and $b^x + 29^y = z^2$, where $b \equiv 2 \pmod{87}$.

Theorem 1. *If $a \equiv 2 \,(mod 66)$ and $a + 1$ is not a perfect square, then the equation $a^x + 22^y = z^2$ has the unique solution $(a, x, y, z) = (2, 3, 0, 3)$, where x, y, z are natural numbers.*

Theorem 2. *If $a \equiv 2 \,(mod 87)$, then the Diophantine Equation $a^x + 29^y = z^2$ has exactly one solution $(a, x, y, z) = (2, 3, 0, 3)$, where x, y, z are natural numbers.*

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7. Computer Science

On the Fidelity of Explanations in Symbolic-Probabilistic-Neural Hybrid System

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Modern artificial intelligence systems increasingly combine three computational paradigms: neural networks, probabilistic inference, and symbolic rules. Existing Explainable AI (XAI) methods have, for the most part, been developed separately for each paradigm. For a hybrid system, however, an important question arises: does the explanation provided for the entire system faithfully reflect its overall behavior, or only that of some of its components? This property, referred to here as explanation fidelity, is investigated using the framework of category theory [1,2].

The proposed framework models the three paradigms by appropriate categories: a category of parameterized functions for the neural component, a category of stochastic channels for the probabilistic component, and a preorder category of predicates for the symbolic component. Explanations themselves form a category, while the mapping from each computational paradigm to the space of explanations is described by functors. Consequently, a hybrid system can be explained in two different ways: either by explaining the composed system directly or by composing the explanations of its individual components.

Fidelity is defined as the degree to which these two explanatory pathways coincide. To quantify it, a distance function is introduced on the space of explanations, and the fidelity of a system is defined as the maximum deviation, over all inputs, between the direct explanation and the composed explanation. This definition admits a categorical interpretation in which the two explanatory paths correspond to the sides of a diagram whose exact commutativity is equivalent to perfect fidelity. At the same time, fidelity can be computed numerically without requiring additional mathematical framework.

Within this framework, the fidelity of a hybrid workflow is defined as the naturality defect of the transformation between the composite functor $F_S \circ F_P \circ F_N$ and the direct explanatory functor G: the supremum, taken over all inputs, of the distance between the two paths of the naturality square,

$$\varphi = \sup_x d\big((F_S \circ F_P \circ F_N)(x), G(x)\big).$$

The case $\varphi = 0$ corresponds to the exact commutativity of the diagram, where the composed explanation coincides with the direct explanation for every input. In this notation, the compositionality property investigated in this work takes the form $\varphi \leq \varphi_N + \varphi_P + \varphi_S$, where φ_N, φ_P, and φ_S denote the naturality defects of the neural, probabilistic, and symbolic components, respectively.

The construction is illustrated using a hybrid system consisting of a small neural network, a probabilistic calibration module, and a rule-based symbolic layer. For this example, the deviation is computed explicitly, and the compositionality property is formulated and experimentally verified: the deviation of the entire system does not exceed the sum of the deviations of its individual components. The paper represents a first step toward moving from a qualitative treatment of explainability in hybrid systems to a quantitative and verifiable one.

Keywords*Explainable Artificial Intelligence (XAI); Hybrid Systems; Fidelity; Category Theory; Functors; Compositionality; Neuro-Symbolic Systems.*

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Categorical models for explainable artificial intelligence

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Recent advances in Artificial Intelligence (AI) have led to the deployment of increasingly sophisticated machine learning models whose internal decision-making mechanisms remain largely opaque. This lack of transparency presents significant challenges for scientific reproducibility, regulatory compliance, ethical accountability, and human trust. Explainable Artificial Intelligence (XAI) has therefore emerged as a fundamental research direction seeking to provide mathematically grounded methods for interpreting and validating intelligent systems. Despite remarkable progress in post-hoc explanation techniques and inherently interpretable models, existing approaches often lack a unified theoretical framework capable of describing explanation processes independently of specific learning architectures.

This paper proposes Category Theory as a universal mathematical language for formalizing explainability in artificial intelligence. Rather than considering explanations as isolated algorithmic artifacts, the proposed framework models learning systems, inference procedures, and explanation mechanisms as compositional categorical structures. Objects represent datasets, feature spaces, latent representations, predictive models, and explanation domains, whereas morphisms formalize learning algorithms, feature transformations, inference operators, and explanation mappings preserving structural properties.

Functorial semantics establish rigorous correspondences between computational and explanatory domains, allowing explanations to be propagated across heterogeneous model architectures while preserving semantic consistency. Furthermore, natural transformations describe the equivalence of multiple explanation strategies operating over different model representations. Universal constructions—including products, coproducts, pullbacks, pushouts, limits, and colimits—provide elegant abstractions for modular learning architectures and compositional explainability.

The proposed categorical framework further integrates symbolic reasoning, probabilistic inference, and deep neural computation within a common algebraic structure. Monoidal categories describe parallel computation and information flow, while adjoint functors characterize bidirectional transformations between prediction and explanation spaces. Such formalization enables rigorous reasoning about transparency, causality, modularity, invariance, and verification of explanation algorithms.

The study demonstrates that categorical semantics establish a mathematically consistent foundation for next-generation Explainable Artificial Intelligence, offering a unified perspective capable of supporting trustworthy, verifiable, and compositional intelligent systems.

Keywords*Explainable Artificial Intelligence; Category Theory; Categorical Semantics; Functors; Natural Transformations; Explainability; Neural Networks; Mathematical Foundations of AI; Trustworthy AI; Compositional Learning.*

Realistic 3D Character Animation Using Rigging and Motion Capture Systems

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In the context of the rapid advancement of the digital entertainment industry, AAA video games, and virtual reality, the demand for realistic 3D character animation has reached unprecedented levels. Although traditional keyframe animation provides complete artistic control, it becomes inefficient for large-scale productions, as it is labor-intensive and difficult to scale. Consequently, motion capture (MoCap) technologies and advanced rigging systems have become industry standards. However, transferring raw MoCap data to character models with different skeletal structures and anatomies often results in significant visual artifacts, such as mesh intersections, foot sliding, and loss of ground contact.

The relevance of this research lies in the need to optimize the workflow for transferring and adapting (retargeting) motion data while ensuring realistic biomechanical behavior. This paper examines modern techniques for realistic 3D character animation through the integration of advanced rigging systems and motion capture technology. It analyzes the workflow required to achieve smooth and natural character motion, demonstrating, through a comparison of traditional animation methods and motion capture-based approaches, the critical role of rigging in the accurate transfer of biomechanical motion. Finally, the study highlights the importance of optimized rigging and control structures in improving production efficiency and addressing the technical challenges associated with animation implementation.

Keywords *3D animation, rigging, motion capture, computer graphics, digital models.*

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Analysis of Explainable Artificial Intelligence (XAI) Methods and Evaluation of Their Applicability in Decision-Making Systems for the Insurance Industry

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The increasing adoption of Artificial Intelligence (AI) in the insurance industry has significantly improved the automation of decision-making processes, including underwriting, fraud detection, claims assessment, and risk prediction, as highlighted by recent studies and regulatory reports on AI governance in insurance [6, 7]. However, the widespread use of complex machine learning models, particularly deep learning techniques, has introduced important challenges regarding transparency, interpretability, accountability, and regulatory compliance [5, 4]. These concerns are especially critical in insurance, where automated decisions directly affect customers and must satisfy legal and ethical requirements.

This paper presents a comprehensive analysis of Explainable Artificial Intelligence (XAI) methods and evaluates their applicability to decision-support systems in the insurance sector. The study reviews the theoretical foundations of XAI and classifies existing explainability techniques according to their scope, including intrinsic and post-hoc methods, local and global explanations, as well as model-specific and model-agnostic approaches [5, 4, 3]. Particular attention is given to widely adopted techniques such as SHAP, LIME, counterfactual explanations, feature importance analysis, partial dependence plots, and surrogate models, which have become standard approaches for interpreting black-box machine learning models [1, 2, 3].

The applicability of these methods is examined in representative insurance scenarios, including fraud detection, claims processing, customer risk profiling, underwriting, and premium pricing. Recent studies have demonstrated the growing importance of explainability for fraud detection and regulatory compliance in insurance environments [8, 7]. Their advantages and limitations are evaluated with respect to explanation quality, computational efficiency, scalability, robustness, regulatory compliance, and user trust, following the evaluation criteria proposed in recent XAI literature [5, 3]. The paper also discusses current challenges related to explanation consistency, fairness, privacy preservation, and the integration of XAI into distributed insurance information systems based on modern software architectures.

The analysis demonstrates that no single XAI technique is universally suitable for all insurance applications. Instead, selecting an appropriate explainability method depends on the underlying machine learning model, the business context, regulatory requirements, and the intended end users. The presented study provides a systematic foundation for integrating trustworthy and explainable AI into next-generation insurance decision-support systems while promoting transparency, accountability, and responsible AI adoption in accordance with recent international recommendations [6, 7].

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Algorithm for Multiple-Precision Long Division

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The paper presents an efficient algorithm for multiple-precision long division. Unlike the classical algorithm proposed by D. Knuth, the suggested method does not require normalization of the dividend and divisor before division and renormalization of the partial remainder during the computation. This simplifies the implementation while preserving the computational efficiency of quotient digit estimation.

The key contribution of the paper is a new formula for estimating the next quotient digit. The estimate is computed from the three highest digits of the current dividend fragment and the two highest digits of the divisor. In contrast to the classical D. Knuth estimate, the denominator is increased by one. This modification changes the nature of the approximation: instead of producing an upper estimate of the true quotient digit, as in Knuth's algorithm, the proposed formula produces a lower estimate. A mathematical proof is given showing that the estimated digit differs from the exact quotient digit by at most one. Consequently, the correct digit is obtained either immediately or after a single increment, eliminating the repeated correction procedures typical of normalized division algorithms.

The paper proves the correctness of the proposed estimate by establishing inequalities that bound the true quotient digit. In particular, it is shown that the computed estimate always satisfies the lower bound, while the next integer already exceeds the admissible interval by less than one digit. Thus, the algorithm guarantees correctness with at most one correction step.

The proposed approach is compared with several well-known multiple-precision division algorithms. Unlike binary-search approaches for quotient digit selection, the new method computes the digit directly using a closed-form expression. Compared with Knuth's algorithm, it avoids normalization while preserving comparable computational complexity.

The implementation is based on representing long integers as arrays of digits in an arbitrary radix. A compact C language implementation is presented in which the core division procedure

consists of approximately twenty executable statements. The use of pointer arithmetic allows the program to remain concise while maintaining high execution speed. The developed algorithm correctly handles divisors of arbitrary length, including the single-digit case, without requiring a separate specialized procedure.

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Use of category theory in formal languages and automata: theoretical and methodological perspectives

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Category theory is one of the most influential mathematical frameworks developed in the twentieth century, providing a unified approach to describing abstract structures and the relationships between them. Over the past decades, its applications have extended far beyond pure mathematics, encompassing theoretical computer science, formal language theory, programming language design, formal verification, and artificial intelligence. Within the context of the course Formal Languages and Automata, category theory offers a rigorous mathematical framework for modeling finite automata, formal grammars, and the transformations between different computational models. The purpose of this paper is to analyze the theoretical foundations of applying category theory to the study of formal languages and automata, highlighting the conceptual and methodological advantages of this approach. The paper examines the relationships between categorical objects and the structures employed in automata theory, the role of morphisms in modeling transformations between automata, and the contribution of functors to describing equivalences among different mathematical categories. Furthermore, it discusses the pedagogical implications of introducing categorical concepts into university-level instruction, arguing that the categorical approach promotes the development of abstract reasoning and enhances the understanding of structural relationships among the fundamental concepts of theoretical computer science. The findings of the theoretical analysis indicate that category theory provides an effective methodological framework for integrating the concepts of formal languages, finite automata, formal grammars, and formal semantics into a coherent conceptual model. Such an approach facilitates both scientific research and the teaching and learning process by offering a unified perspective on the mathematical foundations of computation.

Keywords *category theory, formal languages, finite automata, categories, morphisms, functors, theoretical computer science, teaching methodology.*

Passenger Satisfaction Profiling in Urban Trolleybus Transport Using K-Means Clustering

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This study analyzes passenger satisfaction with urban trolleybus services using survey data from 1,362 respondents [1]. The analysis focuses on four general dimensions: timetable and information availability, bus stop conditions, travel experience, and the trolleybus route network. Socio-demographic and behavioral variables were used only to characterize the resulting passenger groups.

After standardization [2], K-means clustering was applied for solutions ranging from $k = 2$ to $k = 10$. The partitions were evaluated using the Silhouette coefficient, the Calinski–Harabasz index, the Davies–Bouldin index, and inertia. Although different criteria favored different values of k, the three-cluster solution was selected as the most interpretable and practically useful [3].

The first cluster includes 593 respondents (43.54%) with consistently high satisfaction across all dimensions. The second contains 445 respondents (32.67%) who evaluate the trolleybus route network positively but report more moderate satisfaction with information, stops, and travel conditions. The third comprises 324 respondents (23.79%) with relatively low scores across all dimensions.

The results show that trolleybus passengers are heterogeneous in their perceptions and that satisfaction with the route network does not necessarily imply satisfaction with service delivery. The proposed segmentation can support targeted improvements in passenger information, stop infrastructure, travel comfort, and overall service quality. The proposed segmentation can support targeted service improvements and may contribute to integrated sustainable urban mobility planning, including the coordination of public transport with complementary active mobility infrastructure [4].

Keywords*trolleybus transport; passenger satisfaction; clustering; K-means; passenger segmentation; public transport quality; system analysis.*

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The use of category theory in database programming

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Category theory has become one of the most influential mathematical frameworks for describing abstract structures and their relationships, providing a unified formal language applicable to mathematics, theoretical computer science, software engineering, and information systems. The increasing complexity of distributed databases, cloud computing platforms, semantic data integration, and heterogeneous information systems has created the need for rigorous mathematical methods capable of ensuring correctness, consistency, and interoperability throughout the entire data life cycle. This paper investigates the application of category theory to database programming and database schema modeling. The fundamental concepts of objects, morphisms, functors, and natural transformations are employed to formally represent relational database schemas and the relationships between their entities. Within this framework, database tables are interpreted as categorical objects, foreign-key relationships as morphisms, and schema transformations as functorial mappings preserving structural properties. The paper further discusses categorical approaches to database migration, schema evolution, data integration, and query processing. A comprehensive case study involving an online order management system is developed to illustrate the categorical representation of database transformations, the composition of morphisms, and the preservation of semantic consistency through functorial data migration and natural transformations. The obtained results demonstrate that category theory provides a rigorous mathematical foundation for database design, facilitates formal verification of schema transformations, reduces inconsistencies during migration processes, and supports interoperability among heterogeneous information systems. Consequently, categorical methods represent a promising direction for the development of reliable, scalable, and mathematically verified database technologies.

Keywords *category theory, relational databases, categorical modeling, morphisms, functors, natural transformations, database migration, data integration, SQL, interoperability.*

An Interactive Multimodal Framework for the Semantic Augmentation of Cultural Idioms using AI and AR

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The core novelty of this research lies in its **multimodal, cross-modal pipeline** that bridges linguistic decoding with visual representation. Initially, high-performing **Large Language Models (LLMs)**, such as ChatGPT and DeepSeek, are utilized to extract and articulate the complex

figurative meanings of digitized folklore [1]. These semantic explanations are then converted into prompts for **generative AI systems** to create visual metaphors that illustrate the abstract essence of the idioms.

The practical application of this research is realized through a sophisticated **8-step interactive AR pipeline**, representing a significant advancement in educational technology:

1. **Interactive Scanning:** Users trigger the experience by scanning physical cards.
2. **Language Selection:** The system supports multilingual navigation (Romanian, English, Russian, and Spanish).
3. **Thematic Classification:** Exploration is branched into major categories such as *Emotions* or *Cultural Motifs*.
4. **Specific Selection:** Users select specific sub-motifs to activate the corresponding idiomatic content.
5. **Virtual Card Visualization:** The application overlays AI-generated visual metaphors representing both *literal* and *figurative* senses.
6. **Multilingual Mapping:** Displays equivalent expressions in other languages to facilitate "sense-for-sense" translation.
7. **Intercultural Analysis:** Provides deep insights into cultural contexts and associated dominant emotions.
8. **Exploratory Feedback Loop:** Allows seamless return to categories to foster continuous learning.

This framework transforms passive study into an **engaging multisensory experience**, fostering **digital literacy** and **educational inclusion** while ensuring that the intangible cultural heritage remains accessible in the digital era.

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8. Education

Mathematical modeling of communication channels

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Abstract. The increasing complexity of modern communication systems and the rapid development of wireless technologies have intensified the need for mathematical approaches capable of accurately describing the behavior of communication channels. At the same time, higher education requires effective methodologies that connect theoretical mathematical concepts with real-world engineering applications. This study explores the role of mathematical modeling as a fundamental tool for analyzing, interpreting, and teaching communication channel behavior. The research is based on a theoretical and methodological analysis of mathematical models commonly used to represent communication channels, including deterministic and stochastic approaches. The study examines the mathematical principles underlying signal propagation, attenuation, interference, noise, and channel capacity, emphasizing the use of algebraic, probabilistic, and analytical methods for describing communication processes. Particular attention is given to the educational value of mathematical modeling in facilitating students' understanding of abstract concepts through quantitative analysis and problem-solving. The findings demonstrate that mathematical modeling provides an effective framework for predicting communication channel performance, evaluating transmission reliability, and supporting the design and optimization of modern communication systems. Furthermore, integrating communication channel models into mathematics and engineering education enhances analytical thinking, mathematical reasoning, computational skills, and interdisciplinary competence by linking mathematical theory with practical technological applications. The study contributes to the theoretical foundation of communication channel modeling from both scientific and educational perspectives. It highlights the importance of mathematical modeling as a bridge between mathematics, information and communication technologies, engineering, and STEM education, providing a methodological basis for future research and curriculum development in emerging communication technologies.

Keywords *mathematical modeling; communication channels; wireless communications; stochastic models; signal propagation; channel capacity; engineering education; STEAM education; applied mathematics; communication systems.*

Elements of absolute geometry in high school

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The article proposes a reassessment of the geometric content taught in the gymnasium cycle from the perspective of absolute (neutral) geometry, a branch of classical geometry that does not assume the validity of Euclid's Parallel Axiom. By highlighting the common axiomatic foundation — based on the axioms of incidence, order and congruence — the paper demonstrates that numerous theorems studied in school are, in fact, results of absolute geometry. The contributions of the

system of axioms formulated by David Hilbert are analyzed and their direct applications in the demonstration of elementary geometric propositions are discussed. The article also explores the formative valences of introducing this conceptual framework into the gymnasium didactic process: the development of logical thinking, familiarization with the axiomatic structure and facilitating the transition to non-Euclidean geometries. The paper argues that, although it is not explicitly included in the secondary school curriculum, it can be integrated through interdisciplinary themes with the aim of being consciously capitalized on in current lessons, as a solid foundation for a rigorous understanding of geometry. Methodological suggestions are offered for adapting the content to the cognitive level of students, underlining the role of this approach in the formation of an authentic mathematical culture open to theoretical generalization.
Keywords *absolute geometry, Hilbert's axioms, middle school education, axiomatic structure, logical thinking, geometry teaching.*

Enhancing engineering mathematics education through the flipped classroom model

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The rapid digitalization of higher education, the expansion of distance learning, and the challenges faced by Ukrainian universities under martial law require the implementation of innovative teaching approaches that improve student engagement and learning outcomes. One of the most promising pedagogical models is the flipped classroom, which shifts the acquisition of theoretical knowledge to students' independent learning while using classroom time for collaborative problem-solving and practical application of knowledge. The purpose of this study is to examine the pedagogical potential of the flipped classroom model in teaching higher mathematics at technical universities and to identify its advantages for developing professional competencies of future engineers. The flipped classroom model is based on three interconnected stages. Before class, students independently study theoretical material using digital educational resources, including video lectures, interactive notes, and methodological materials. During synchronous classes, students solve complex mathematical and engineering-oriented problems, participate in discussions, receive feedback from the instructor, and collaborate in small groups. After class, additional practice-oriented assignments help consolidate the acquired knowledge and encourage further independent learning. The implementation of this model transforms the role of classroom instruction. Instead of spending valuable class time on presenting theoretical concepts, instructors focus on supporting students in applying mathematical methods to real engineering problems. Such an approach promotes active learning, improves conceptual understanding, and increases students' motivation and responsibility for their own educational achievements. Despite its advantages, successful implementation of the flipped classroom requires overcoming several challenges. The most significant difficulty is ensuring students' adequate preparation before synchronous sessions. To address this issue, a combination of motivational strategies is recommended, including pre-class online quizzes, continuous assessment, interactive multimedia resources, gamification elements, and systematic instructor feedback. The gradual introduction of the flipped classroom model also facilitates students' adaptation to this educational approach and contributes to the development of self-directed learning skills. The use of digital learning management systems such as Moodle or Google Classroom significantly enhances the organization of the educational process by providing centralized access to learning materials, assessment tools, and communication channels. These technologies increase learning flexibility,

which is particularly important under conditions of distance education and unstable access to educational resources. The analysis of pedagogical experience and recent research demonstrates that the flipped classroom positively influences students' academic performance, engagement, critical thinking, analytical reasoning, problem-solving abilities, digital literacy, communication skills, and self-organization. Moreover, it creates favorable conditions for integrating mathematical knowledge with professionally oriented engineering tasks, thereby strengthening the practical relevance of higher mathematics education. In conclusion, the flipped classroom model represents an effective strategy for modernizing mathematics education at technical universities. Its implementation promotes active student participation, improves learning outcomes, and supports the development of competencies required for future engineering professionals. The model can therefore be considered a valuable tool for enhancing the quality and practice-oriented character of higher mathematics education in contemporary higher education.

Keywords*flipped classroom; higher mathematics; technical university; distance learning; digital educational technologies; practice-oriented learning.*

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Aplicații ale produsului scalar la determinarea valorilor de extrem ale unor funcţii

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Determinarea soluțiilor unor probleme de extreme condiționate poate fi efectuată mult mai simplu și mai efectiv, dacă vom aplica produsul scalar al vectorilor și proprietățile lui. Pentru studiul exemplelor ce vor urma este importantă următoarea teoremă:

Theorem 1. *Fie dată funcția*

$$F = \sum_{k=1}^{n} a_k f_k \left(x_1, x_2, \ldots, x_n\right) \tag{1}$$

Fie că este adevărată următoarea egalitate

$$\sum_{k=1}^{n} b_k^2 f_k^2 \left(x_1, x_2, \ldots, x_n\right) = d^2, \tag{2}$$

în care a_k sunt constante arbitrare, iar d și b_k sunt constante pozitive. Atunci are loc:

$$|F| \leq dp, \text{unde } p = \sqrt{\left(\frac{a_1^2}{b_1^2} + \frac{a_2^2}{b_2^2} + \ldots + \frac{a_n^2}{b_n^2}\right)}$$

Demonstrație. Considerăm vectorul $\vec{u} = (b_1 f_1, b_2 f_2, \ldots, b_n f_n)$ și vectorul $\vec{v} = \left(\frac{a_1}{b_1}, \frac{a_2}{b_2}, \ldots, \frac{a_n}{b_n}\right)$. Produsul scalar al acestor vectori calculat în coordonate va fi egal cu funcția F din (1). Notând unghiul dintre acești vectori prin φ și aplicând definiția produsului scalar, vom obține:

$$\lceil F \rceil = \lceil \vec{u} \cdot \vec{v} \rceil \leq |\vec{u}| \cdot |\vec{v}| |\cos \varphi| \leq d \cdot p.$$

Folosind această teoremă vom studia următoarele exemple.

Exemplul 1. Fie $\vec{u}$ un vector arbitrar ce formează cu axele de coordonate OX, OY și OZ unghiuri egale cu α, β și φ corespunzător. Să se afle valoarea maximă și cea minimă a expresiei $F = a \cdot \sin\alpha + b \cdot \sin\beta + c \cdot \sin\varphi$, unde a, b, c sunt constante reale.

Rezolvare: Considerăm vectorii: $\vec{u} = (\sin\alpha, \sin\beta, \sin\varphi)$ și $\vec{v} = (a, b, c)$. Atunci produsul lor scalar va fi egal cu funcția F. Din identitatea cosinusurilor directoare ale unui vector avem, că $\cos^2\alpha + \cos^2\beta + \cos^2\varphi = 1$.

De aici $1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\varphi = 1$. De unde obținem identitatea necesară: $\sin^2\alpha + \sin^2\beta + \sin^2\varphi = 2$.

Atunci, conform teoremei, $d = \sqrt{2}$, $p = |\vec{v}| = \sqrt{(a^2 + b^2 + c^2)}$ și $maxF = dp = \sqrt{2\,(a^2 + b^2 + c^2)}$, $minF = -dp = -\sqrt{2\,(a^2 + b^2 + c^2)}$. Ușor se observă, că aceste valori vor fi obținute în cazul de maxim (minim) din următoarele egalități: $\sin\alpha = \frac{+(-)\sqrt{2}a}{p}$; $\sin\beta = \frac{+(-)\sqrt{2}b}{p}$; $\sin\varphi = \frac{+(-)\sqrt{2}c}{p}$.

De exemplu, dacă $a = 1$, $b = -\sqrt{2}$, $c = 1$ atunci $max(min)F = \sqrt{8}(-\sqrt{8})$. $\sin\alpha = \frac{+(-)\sqrt{2}}{2}$; $\sin\beta = -(+)1$; $sin\varphi = \frac{+(-)\sqrt{2}}{2}$. Valoarea maximă (minimă) a expresiei F va fi asigurată pentru vectorul ce formează cu axele de coordonate următoarele unghiuri: $\alpha = +(-)\frac{\pi}{4}$, $\beta = +(-)\frac{\pi}{2}$, $\varphi = +(-)\frac{\pi}{4}$.

Exemplul 2. Să se afle valoarea maximă și cea minimă a expresiei $F = ax + by + cz$ cu condiția $4x^2 + 16y^2 + z^2 = 16$.

Rezolvare: Considerăm vectorii: $\vec{u} = (2x, 4y, z)$ și $\vec{v} = \left(\frac{a}{2}, \frac{b}{4}, c\right)$. Produsul lor scalar va fi egal cu funcția F. Și $d = 4$, iar $p = |\vec{v}|$. Soluția acestei probleme este:

$$max(min)F = +(-)dp, \quad \text{unde} \quad dp = \sqrt{(4a^2 + b^2 + 16c^2)}.$$

Valoarea maximă(minimă) a expresiei F va fi obținută pentru următoarele valori ale variabilelor: $x = \frac{+(-)4a}{dp}$, $y = \frac{+(-)b}{dp}$, $z = \frac{+(-)16c}{dp}$, $dp = \sqrt{(4a^2 + b^2 + 16c^2)}$.

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Didactic approaches to teaching the course "integrated security systems"

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Cybersecurity methods encompass a wide range of information protection tools. Software, hardware, and physical information protection tools are considered formal methods of cybersecurity.

These include integrated security systems. Teaching these course or module is fraught with certain challenges: the lack of specialized technical equipment at the educational institution, as well as a shortage of teaching materials and visual aids. To support the course's methodological support, the course goals and objectives have been formulated, teaching methods and formats have been defined, and the course content and structure have been developed, covering theoretical sections, practical topics, independent work and project activities.

Keywords*integrated security systems, didactic approaches, ICT, cybersecurity*

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Teaching Risk Management with Card Problems

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Risk management plays an essential role in economics, finance, engineering, and everyday decision making, yet it rarely appears explicitly in school mathematics. This article presents several card problems that allow gifted middle and high school students to discover fundamental principles of risk management using only elementary probability, logical reasoning, and strategy.

In the first problem, students investigate when to place a single bet while observing the colors of previously revealed cards. Although intuition suggests that waiting for a favorable imbalance should create an advantage, students discover that every stopping strategy has exactly the same expected value. This provides a natural introduction to fair games, conditional probability, and the surprising limitations of additional information.

The second problem considers repeated betting with adjustable stake sizes. Here the objective is no longer to maximize expected winnings, but to maximize the amount that can be guaranteed regardless of the order of the remaining cards. Students are led to the idea of betting only a carefully chosen fraction of their current bankroll rather than risking everything at once. The optimal strategy illustrates how diversification and controlled risk can outperform aggressive betting, while introducing concepts closely related to hedging and optimal capital allocation.

The article discusses classroom implementation and possible extensions suitable for mathematical circles and Olympiad enrichment programs. These problems provide an accessible setting in which students experience how mathematical reasoning can guide decision making and illuminate fundamental ideas about expected values of possible outcomes and the management of risk.
Keywords *mathematical competence, curricular enrichment, risk management, card games.*

Evaluating Merge EDU for mathematics learning through augmented reality experiences

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The integration of Augmented Reality (AR) technologies into mathematics education represents an essential research direction for supporting the construction of mental representations of abstract concepts and developing spatial thinking. This article evaluates the potential of the Merge EDU platform from two complementary perspectives: analyzing the predefined AR activities available in the ecosystem and exploring the process through which teachers can design their own interactive learning scenarios. The methodology used is exploratory, grounded in key pedagogical criteria such as three-dimensional visualization capability, level of interaction, design flexibility, and the accessibility of authoring tools for teachers. To this end, a conceptual architecture structured into five iterative stages is proposed, ranging from the teacher defining objectives to the student actively constructing knowledge, integrated within the platform's cloud infrastructure. The results highlight that Merge EDU facilitates learning through exploration and direct manipulation, reducing the gap between mathematical abstraction and visual representation. Furthermore, the platform transforms the teacher's role from a passive technology consumer into an active designer of educational experiences. However, effective integration remains contingent on overcoming logistical challenges and developing the digital and methodological skills of educators. The paper concludes with future research prospects oriented towards the empirical evaluation of users' cognitive experience through psychometric instruments designed to measure how the use of Extended Reality systems extends or supports the user's cognitive abilities during immersion.
Keywords *Augmented Reality, AR, Merge EDU, mathematics education, spatial representations, cognitive evaluation.*
Acknowledgments. This study was conducted within the project "Methodology of implementing ICT in the process of studying real sciences from the perspective of the STEAM concept and Artificial Intelligence", project number 040101 (Ion Creangă State Pedagogical University of Chișinău), and within the framework of the research and innovation project "New extended reality technology for cognitive augmentation based on the superimposition of interactive experiences", registered in the State Register of Projects in the field of Science and Innovation of the Republic of Moldova, project code 25.80013.5007.60ROMD.

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Conceptual delimitation between aerial algebra and aerial geometry

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The rapid advancement of unmanned aerial systems, autonomous platforms, and intelligent aerial technologies has generated the need for new mathematical frameworks capable of describing and analyzing aerial phenomena. While the concepts of Aerial Algebra and Aerial Geometry have recently been introduced as emerging branches of aerial mathematics, their theoretical boundaries and research scopes have not yet been systematically defined. This study addresses this conceptual gap by establishing a rigorous distinction between the two disciplines. The research employs a conceptual and methodological analysis based on comparative, structural, and functional approaches. The study identifies the object of investigation, theoretical foundations, methodological principles, and application domains of each discipline. The results demonstrate that Aerial Algebra primarily investigates algebraic structures, symbolic representations, optimization models, communication processes, coding mechanisms, and computational algorithms governing aerial systems. In contrast, Aerial Geometry is concerned with the mathematical description of spatial structures, geometric transformations, trajectories, positioning, navigation, three-dimensional modeling, and the spatial organization of aerial environments. Although both disciplines contribute to the mathematical analysis of aerial systems, they address fundamentally different scientific questions while remaining methodologically complementary. The proposed conceptual framework provides the first systematic delimitation between Aerial Algebra and Aerial Geometry and positions both disciplines within the broader framework of aerial mathematics. This theoretical foundation contributes to

the development of a coherent research agenda for interdisciplinary studies involving mathematics, autonomous aerial systems, robotics, artificial intelligence, aerospace engineering, and STEM education. Furthermore, the proposed classification offers a basis for curriculum development, mathematical modeling, and future theoretical and applied investigations in emerging aerial technologies.

Keywords*aerial algebra; aerial geometry; aerial mathematics; conceptual framework; mathematical modeling; unmanned aerial systems; autonomous navigation; optimization; spatial modeling; interdisciplinary mathematics.*

Computing the sum of certain numerical series

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Numerical series arise in numerous areas of both theoretical and applied mathematics, serving as an essential tool in mathematical analysis, numerical computation, and modeling problems. Determining the sum of a convergent numerical series is a classical problem, while the existence of a finite limit of its partial sums allows the series to be associated with a specific numerical value ([1], [2]). This paper presents, through illustrative examples, efficient methods for computing the sums of convergent numerical series, with particular emphasis on the method of undetermined coefficients ([3], [4]) and a method based on partial differences.

The method of partial differences is based on the following statement:

Proposition 1. *Let the terms of the series* $\sum_{n=1}^{\infty} a_n$ *be of the form* $a_n = b_{n+1} - b_n$ *and suppose that the finite limit* $\lim_{n\to\infty} b_n = b$ *exists. Then the series* $\sum_{n=1}^{\infty} a_n$ *is convergent and has the sum* $\sum_{n=1}^{\infty} a_n = b - b_1$.

The presented methods enable the rapid and efficient computation of the sums of convergent series, particularly when the general term is a rational function. The method of undetermined coefficients provides a powerful tool for the decomposition of rational fractions, whereas the method of partial differences yields an elegant transformation into telescoping series. These techniques are well justified in numerous applications of mathematical analysis.

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On curricular enrichment from the perspective of mathematical modeling competence training

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We examine the possibilities for enriching the Mathematics Curriculum from the perspective of developing mathematical modelling competence, which is regarded as an essential competence in modern competency-based education. It emphasizes that mathematics should no longer be viewed solely as a discipline focused on acquiring concepts and computational algorithms, but also as a means of fostering students' analytical and critical thinking, as well as their ability to investigate and solve real-life problems through mathematical models.

The study has a theoretical and practical character and is based on an analysis of the Mathematics Curriculum for pre-university education, as well as a review of the relevant literature in mathematics education and mathematical modelling. The curriculum analysis aimed to identify the curricular subjects that support the development of mathematical modelling competence, assess the coherence between competences, content, and learning activities, and highlight the aspects requiring curricular enrichment.

A set of curriculum enrichment criteria was formulated to support the design of learning activities focused on mathematical modelling. These criteria include the relevance of the content for different groups of learners, the authenticity of problem situations, the interdisciplinary nature of the contexts, the mathematical potential of the selected content, the continuity of competence development, the involvement of higher-order cognitive processes, the integration of digital technologies, and the possibility of practical or simulated testing and validation of the proposed solutions.

The article emphasizes that authentic, interdisciplinary tasks enhance students' motivation and promote the application of mathematics in real-world contexts. It also highlights the importance of organizing modelling activities progressively, from simple to more complex situations, and of encouraging the use of multiple solution strategies to develop flexibility of thinking, creativity, and reasoning skills. The integration of digital technologies supports the investigation of phenomena, data visualization, and the construction and validation of mathematical models.

The authors argue that enriching the Mathematics Curriculum according to the proposed criteria can contribute to the development of mathematical modelling competence and to preparing students to apply mathematical knowledge effectively in analyzing and solving real-world problems. Furthermore, the proposed framework provides methodological guidelines for the design of educational resources and serves as a foundation for future applied research in mathematics education.

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Integration of UX/UI design in the design of an online registry

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In the context of the accelerated digitalization of administrative processes, the development of efficient online platforms has become a critical priority. The success of a digital registry depends not only on its technical functionality but also on the quality of the user experience (UX) and the user interface (UI). This paper explores the process of implementing UX/UI design principles in the development of online registries, highlighting their role in creating an intuitive and engaging user experience. The study examines essential UI/UX concepts, methodologies, and specific design tools, detailing the fundamental stages of the workflow: user research, the development of wireframes and mockups, and the coherent selection of visual elements, such as typography, color palettes, and animations.

Furthermore, the author addresses strategies for optimizing information architecture and interface accessibility. By comparing traditional models with modern web-based solutions, the study demonstrates how a rigorous UX/UI approach can reduce human error and streamline administrative workflows. The paper emphasizes that a well-conceived design not only ensures intuitive interaction but also directly influences platform performance and search engine visibility. Ultimately, this study provides a methodological framework for implementing intuitive interfaces, representing a necessary step toward the sustainable digitalization of public services

Keywords *UX/UI design, user experience, prototyping, collaboration, adaptation, Figma, animations.*

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Solving Classical Probability Problems Using R Programming

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Probability concepts can often feel abstract to students encountering them for the first time. This paper demonstrates how foundational, classical probability problems—such as coin tosses,

dice rolls, and urn extractions—can be enhanced through simple implementations in the R programming language. The methodology details classroom activities designed to make standard probability exercises more tangible and engaging. Results show that integrating simple code helps clear up common student misconceptions and reinforces core logical thinking. In conclusion, this approach offers educators a practical framework to combine traditional mathematics with introductory programming, turning classic text-book problems into interactive learning experiences.

Keywords *Classical probability, R programming, introductory STEM, educational software, simulation*

Methods for the evaluation of certain finite sums

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The computation of finite sums is an important area of mathematical analysis, as it enables the derivation of compact formulas for partial sums and facilitates the study of the convergence of numerical series. This paper presents two general methods for computing finite sums, both based on transforming the general term of the sum into a difference of values of an auxiliary function ([1]–[5]).

The first method assumes the existence of a function $(S(k))$ such that the general term of the sum can be expressed in the form $(S(k+1)-S(k))$ or $(S(k)-S(k+1))$. Under these conditions, the finite sum becomes a telescoping sum, in which most intermediate terms cancel each other. Consequently, a simple formula for the sum of the first (n) terms is obtained, involving only the values of the auxiliary function at the endpoints of the summation interval.

The second method generalizes this idea to the case where the difference of the auxiliary function is evaluated over an arbitrary step (m), that is, when the general term has the form $(S(k+m)-S(k))$. In this case, the finite sum can be expressed as the difference of two sums involving the auxiliary function, making it possible to evaluate broader classes of finite sums that cannot be handled directly by the classical telescoping technique.

The proposed methods are applicable to a wide variety of mathematical expressions, including those involving rational, logarithmic, trigonometric, and factorial functions. Owing to their generality, these methods provide an efficient tool for both mathematical analysis and the study of numerical and functional series.

In conclusion, these methods simplify proofs, reduce computational effort, and contribute to a deeper understanding of series convergence. They are therefore of both theoretical and practical significance.

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Applications of the Cauchy-Bunyakovsky-Schwarz inequality

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The Cauchy–Bunyakovsky–Schwarz (CBS) inequality is one of the fundamental results in mathematics, playing a central role in linear algebra, mathematical analysis, and numerous other areas of the exact sciences. Owing to its generality and broad applicability, this inequality provides an efficient tool for proving mathematical relations, estimating expressions, and solving optimization problems.

This paper presents a proof of the CBS inequality based on the geometric interpretation of the scalar product of two vectors in Euclidean space. By exploiting the relationship between the scalar product, the norms of the vectors, and the angle between them, the classical form of the inequality for three-dimensional vectors is derived. The result is then generalized to vectors with an arbitrary number of components, leading to the general form of the Cauchy–Bunyakovsky–Schwarz inequality. The conditions under which equality holds are also discussed. ([1]–[6])

Based on this inequality, several classes of mathematical problems are investigated. The proposed approach provides a straightforward method for proving algebraic and trigonometric inequalities, establishing estimates for expressions involving radicals, and determining the maximum or minimum values of algebraic expressions and functions. Moreover, the equality condition in the CBS inequality serves as an effective criterion for solving equations and optimization problems, significantly reducing the amount of computation required.

An appropriate choice of vectors, together with a direct application of the inequality, leads to concise and elegant solutions, avoiding complicated algebraic manipulations and the use of more sophisticated techniques.

The results presented highlight the universal character of the Cauchy–Bunyakovsky–Schwarz inequality and demonstrate its effectiveness in solving a broad range of mathematical problems. The methods discussed can also be extended to other areas of mathematics, including mathematical analysis, geometry, optimization theory, and the study of functions, making them valuable tools for both research and teaching.

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Applications with a circle and several points solved in Maple

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A method for approaching some analytical geometry problems with the help of the Maple environment, related to some applications with a circle and several points in the two-dimensional Euclidean space, problems that classically solved involve the application of the same formula several times and a lot of time would be lost is presented. First, the classical solution method is presented (for some applications without complete solution), using the application of the formulas presented in the Analytical Geometry or Computational Geometry courses, then, the same problems are solved with the help of the Maple environment.
Keywords*point, straight line, circle, Maple*

Inclusion and Artificial Intelligence as a help for school educators

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With each passing day, the number of children with special educational needs is growing very fast. Today, children with special educational needs are in almost every classroom, but teachers in schools are not professionally ready. How to conduct lessons in a way that is accessible to both typical and inclusive children. As a help, we can turn to artificial intelligence to make lessons simpler and more accessible. For some, information is more accessible in auditory format, and for others in visual format. Artificial Intelligence helps us prepare lessons faster and in different versions. However, methods for checking correctness must also be developed.
Keywords*inclusion, artificial intelligence.*

Ethical integration of artificial intelligence in education: challenges and issues

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This article examines the challenges and issues related to the ethical integration of Artificial Intelligence (AI) in education, focusing on the STEAM perspective and the specific context of

the Republic of Moldova. Building on international reference frameworks (The UNESCO Recommendation on the Ethics of AI and the European AI Act), the study identifies several structural problems at national level: the absence of institutional units dedicated to supporting and monitoring AI use, the lack of national normative acts, the absence of validated pedagogical methodologies, the deficit of AI competencies among teaching staff and learners, and the uncontrolled emergence of self-proclaimed AI "experts." The article also examines challenges related to academic assessment under widespread AI use, academic integrity and plagiarism, personal data protection, and access inequalities generated by insufficient research funding. To outline applicable solutions, four European models of institutional AI governance (Estonian, Dutch, German, and Swiss) are comparatively analyzed. The article provides a conceptual framework useful for policymakers, educators, and researchers involved in the ethical modernization of the educational process.
Keywords *Artificial Intelligence; ethics; STEAM education; academic integrity; institutional governance; Republic of Moldova.*

The Dual Role of Artificial Intelligence in Disinformation: From Amplification to Detection and Regulation

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This analysis examines the dual role of artificial intelligence in relation to the phenomena of disinformation, manipulation, propaganda, and deepfakes in the online environment, addressing both how generative AI amplifies these phenomena and how the same technology can be used to detect and counter them. The study includes a clear conceptual delimitation between disinformation, misinformation, malinformation, propaganda, information manipulation, and deepfakes, which are not synonyms but distinct phenomena operating at different levels (content, mechanism, strategy, format), within a framework grounded in the reference literature of the Council of Europe and in European regulation on artificial intelligence. From this standpoint, the period 2024–2026 is identified as a critical juncture: the mass accessibility of generative AI, at near-zero cost, coincides with the densest electoral calendar in recent history. In analyzing AI as an amplifier of disinformation, the authors document, with concrete and verified examples from the Republic of Moldova, Romania, Ukraine and the United States, four key mechanisms: content production at scale, coordinated amplification through account networks and emotional microtargeting, governance failures of major generative models, and, not least, the structural cost asymmetry between attack and defense. The article also examines AI as a tool for countering disinformation, highlighting solutions corresponding to the previously described mechanisms, proposed within the MIDECOD project through the architecture of the RealInfo system (linguistic analysis, emotional classification, coordination detection through semantic similarity) and the pedagogical model of information and media literacy, while also pointing out the remaining open directions for development, particularly regarding synthetic visual content.
Keywords *Artificial Intelligence, LLM, deepfake, voice cloning, synthetic identity, disinformation.*

On mathematics instruction technology in high school based on elements of mathematical modeling

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It should be noted that word problems represent a crucial pillar of mathematics at the middle and high school levels, as they allow us to see the connection between theoretical mathematics and everyday life. Instructional technology based on mathematical modeling intervenes precisely where students face the greatest difficulties, transforming the problem statement into a logical process followed in clear steps. By implementing this instructional approach, the teacher moves beyond using simple, formulaic templates and instead teaches students how to think, analyze, and translate everyday problems into mathematical language. This article describes an instructional approach based on mathematical modeling, using concrete examples suitable for mathematics lessons.

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The Problem of Squaring the Circle: Historical Development and Mathematical Perspectives

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This paper examines the historical and mathematical development of the problem of squaring the circle, one of the most celebrated problems of classical geometry. Starting from the original meaning of quadrature as the measurement of areas and its practical use in land surveying, the paper outlines the emergence of the problem in Ancient Greece and the numerous attempts to construct, using only an unmarked straightedge and a compass, a square having the same area as a given circle. Particular attention is paid to the relationship between this problem and the number π, highlighting the various approximations of its value used throughout history and in different cultural contexts. The paper also discusses the decisive contribution of Ferdinand von Lindemann,

who proved in 1882 that π is a transcendental number, thereby establishing the impossibility of solving the problem exactly by means of classical Euclidean constructions. Finally, several methods for approximating the quadrature of the circle are presented, illustrating how accurate approximate solutions can be achieved despite the impossibility of an exact construction and emphasizing the mathematical significance of this enduring geometric challenge.

Game-based learning in teaching programming: rpg-type educational platforms

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The growing interest in interactive learning methods has led to the development of a growing number of educational platforms that use game mechanics to develop programming skills. Recent research in the field of educational technologies highlights that multimedia resources [1-4], immediate feedback and gamification elements contribute to increasing student motivation, engagement and performance, especially in introductory programming courses [5]. In this context, Role-Playing Game (RPG) platforms provide a learning environment in which solving programming tasks is integrated into interactive scenarios based on missions, difficulty levels and rewards [6-9]. Based on the analysis of the specialized literature on the efficiency of interactive digital resources and Game-Based Learning approaches in computer science education, a comparative analysis of representative platforms for learning programming, such as boot.dev [6], Codédex [7], CheckiO [8], CodeCombat [9] and others, is developed from the perspective of supported programming languages, game mechanics, pedagogical progression, feedback provided, functionalities for teachers and access models. The results highlight that platforms based on RPG and Game-Based Learning mechanics represent effective tools for developing, in students of Computer Science study programs, algorithmic thinking, problem-solving skills and motivation for learning programming, being particularly useful in the introductory stages of training, such as the Fundamentals of Programming course (year I, sem. I, 6 ECTS) or Introduction to the Python programming language (year I, sem. II, 4 ECTS). At the same time, the analysis emphasizes the need to integrate them into a balanced teaching approach, alongside practical activities and authentic programming projects. Keywords: digital pedagogy, digital education, digital skills, digital environment, conceptual model.

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Developing critical thinking in high school students through the formation of mathematical modeling competence

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In the context of education oriented toward the development of key competences, fostering critical thinking has become one of the essential goals of mathematics education. From this perspective, mathematical modeling competence plays a fundamental role, as it facilitates the transfer of mathematical knowledge to real-life contexts and contributes to the development of students who are able to analyze, justify their reasoning, and make well-founded decisions. Mathematical modeling involves a complex process of identifying and analyzing real-world situations, formulating hypotheses, constructing mathematical models, solving them, interpreting the results, and validating the proposed solution. Progressing through these stages promotes the development of critical thinking by engaging students in investigation, reasoning, reflection, and evaluation.

The paper aims to highlight the relationship between the development of critical thinking and the formation of mathematical modeling competence among high school students, while also identifying methodological approaches to organizing the teaching and learning process in ways that support the simultaneous development of these competences. The mathematical modeling process creates meaningful opportunities for developing critical thinking through the formulation of open-ended problems, the selection of relevant information, the choice of appropriate solution strategies, and the justification of the obtained results.

The study is based on an analysis of relevant literature and a logical-didactic analysis of the curricular content. In our research, we adopt the definition proposed in [1], according to which mathematical modeling competence encompasses: knowledge of the mathematical modeling process; skills to analyze the assumptions and properties of mathematical models, to evaluate their

applicability and validity, and to engage in active modeling within given contexts, including the mathematization of real situations and the application of mathematics in non-mathematical domains; self-regulatory metacognitive abilities that support monitoring and guiding the modeling process; dispositions, such as willingness and a collaborative attitude, that facilitate the successful achievement of modeling objectives. In this regard, mathematical modeling goes beyond being merely a method of applying mathematical knowledge, becoming instead a powerful tool for developing logical reasoning and decision-making skills in complex situations.
From a didactic perspective, the development of critical thinking is supported by using active learning strategies such as inquiry-based learning, problem-based learning, case studies, interdisciplinary projects, and the integration of digital technologies. These approaches provide students with opportunities to explore real-life situations, critically analyze available information, and construct mathematical models that are relevant for interpreting phenomena from various fields. Consequently, the teacher assumes the role of a facilitator of learning, guiding students toward reflection, argumentation, and the validation of their own problem-solving processes.
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Teaching higher mathematics through english: challenges and learning outcomes in engineering education

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The internationalization of higher education and the integration of Ukraine into the European educational and scientific area have significantly increased the importance of bilingual education in engineering training. English is no longer viewed solely as a foreign language but increasingly serves as a medium of instruction that enables students to access international educational resources, participate in academic mobility programmes, and develop professional communication skills. Consequently, bilingual teaching has become one of the promising approaches to improving the quality and competitiveness of engineering education. Teaching higher mathematics in English is a particularly demanding task because students must simultaneously master mathematical concepts and acquire professional terminology in a foreign language. The effectiveness of this approach depends on several pedagogical and organizational factors, including students' initial English proficiency, methodological support, instructors' experience, and the adaptation of teaching materials to national educational standards. This study aims to identify the main challenges of bilingual teaching of higher mathematics for engineering students and to determine whether learning mathematics through English influences students' mastery of subject content. The research was conducted at Zaporizhzhia Polytechnic National University among first-year students majoring in

Power Engineering, Electrical Engineering and Electrical Mechanics. The experiment involved students from both English-medium and Ukrainian-medium groups.. Both groups studied according to the same curriculum and completed identical assessment tasks; the only substantial difference was that English-medium students attended additional English language classes and studied higher mathematics entirely in English.

The pedagogical observations demonstrated that bilingual instruction creates several specific challenges. First, many first-year students have insufficient English proficiency to perceive complex mathematical explanations immediately. This often requires instructors to simplify explanations or spend additional time clarifying terminology and concepts. Second, teaching in English proceeds at a slower pace than teaching in the native language, reducing the amount of material that can be covered during scheduled classroom hours. Third, instructors have to provide Ukrainian equivalents of mathematical terminology to ensure that students are able to use professional vocabulary correctly in both languages. Furthermore, differences between Ukrainian and international approaches to mathematics education make it impossible to rely exclusively on authentic English-language textbooks. Therefore, specially designed methodological materials, including bilingual lecture notes, workbooks, calculation assignments and multilingual mathematical dictionaries, become an essential component of bilingual instruction. Despite these challenges, bilingual teaching offers considerable educational advantages. Students continuously improve their English language competence while studying mathematics, acquire professional engineering vocabulary, gain easier access to international scientific literature, and become better prepared for academic mobility, international Master's programmes, internships, and standardized English examinations such as IELTS, TOEFL, TOEIC, and GRE. inations such as IELTS, TOEFL, TOEIC, and GRE.

To evaluate whether bilingual instruction influences academic achievement, students' examination results in higher mathematics were statistically analysed. To verify the research hypothesis, the results of both groups were analysed using the Z-test for independent samples. The English-medium sample consisted of 46 observations with a mean score of 71.78 and a variance of 65.08, whereas the Ukrainian-medium sample included 80 observations with a mean score of 69.36 and a variance of 28.38. At the significance level $\alpha = 0.05$, the empirical value of the test statistic ($Z = 1.8192$) was lower than the critical value ($Z = 1.96$). Therefore, there were no statistical grounds to reject the null hypothesis concerning the equality of the mean scores. The obtained results indicate that the observed differences between the two groups are attributable to random variation rather than to the language of instruction.

The findings demonstrate that English-medium instruction in higher mathematics does not reduce students' academic achievement when appropriate methodological support is provided. Therefore, bilingual instruction may be considered an effective approach for teaching not only mathematics but also other fundamental and professional engineering disciplines. These results support the further expansion of English-medium instruction in Ukrainian technical universities and provide a basis for future research on the development of bilingual mathematical competence and effective instructional strategies. **Keywords** *higher mathematics, bilingual instruction, English Medium Instruction (EMI), bilingual mathematics education, subject-oriented bilingual teaching/learning models.*

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Integrating cognitively oriented strategies in university mathematics education

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In university mathematics education, students frequently demonstrate procedural proficiency while experiencing persistent difficulties in solving non-routine problems and transferring mathematical knowledge to unfamiliar contexts. This indicates that mathematical learning often remains predominantly algorithmic, with insufficient development of conceptual understanding. This study proposes a unified didactic framework that integrates three complementary perspectives from contemporary cognitive research: Variation Theory (Marton, Tsui [1]), Cognitive Load Theory, with particular emphasis on worked examples (Sweller et al. [2]), and the concept of Productive Failure (Kapur [3]). The proposed framework is based on the assumption that these approaches should be viewed not as competing instructional paradigms but as complementary components of a coherent learning process. Variation Theory facilitates concept formation through systematic variation and the identification of critical mathematical features. Cognitive Load Theory supports efficient learning by combining explicit instruction with worked examples and the gradual withdrawal of instructional guidance. Productive Failure promotes meaningful learning by encouraging students to engage in challenging problem solving before formal instruction, thereby activating prior knowledge and preparing them for deeper conceptual understanding. From the perspective of cognitive psychology, Anderson's distinction between declarative and procedural knowledge provides an additional theoretical rationale for the proposed framework, explaining how conceptual understanding is gradually transformed into procedural fluency through guided practice [4]. Based on this theoretical foundation, a four-stage instructional sequence is proposed: (1) exploratory problem solving without prior explanation; (2) concept formation through structured variation of mathematical examples; (3) guided analysis of worked examples; and (4) independent problem solving with progressively reduced instructional support. Unlike approaches that employ these theories independently, the proposed framework defines their pedagogically meaningful sequence within a single instructional cycle for university mathematics. The framework is designed for practical implementation in higher mathematics education and can be applied to calculus, linear algebra, analytical geometry, and differential equations. The proposed integration demonstrates how complementary cognitive approaches can be combined into a coherent instructional model that promotes both conceptual understanding and procedural fluency. The framework offers instructors a theoretically grounded and practically applicable approach to organizing mathematics instruction in higher education. Keywords: mathematics education, university mathematics, variation theory, cognitive load theory, productive failure, worked examples, conceptual understanding.

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Enhancing Geometry Education through Augmented Reality: A Deep Learning Approach

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This paper discusses the development and integration of Augmented Reality (AR) applications within the research subprogram "Information Systems based on Artificial Intelligence" (SIBIA), aimed to transform geometry education. This initiative employs the reality-virtuality continuum to create immersive and interactive learning environments that bridge the gap between abstract mathematical concepts and practical application.

This research builds upon an initial iteration within the SIBIA framework that focused on the intersection of technical AR challenges and pedagogical learning styles. Drawing on two years of development and empirical testing [1], the project previously explored the optimization of marker recognition—improving tracking ratings from 2-3 to 4-5 stars—and addressed environmental factors such as surface reflectivity by advocating for matte-finished artifacts.

Furthermore, the initial phase established a robust foundation in VAK (Visual, Auditory, Kinesthetic) learning styles, creating specialized scenarios like the "Pi Symphony" to cater to diverse pupil needs. The current stage extends these findings into a comprehensive geometry application for 6th-grade pupils. This extension transitions from basic visualizations to an interactive ecosystem where pupils manipulate 2D and 3D objects, apply geometric formulas to real-world problems, and benefit from AI-driven personalized learning paths.

Designed for ease of use, the software employs an intuitive interface that ensures learners can navigate seamlessly between different geometric modules and definitions.

By integrating AI-driven personalization and high-fidelity AR markers, the SIBIA project demonstrates how emerging technologies can be mplemented in educational institutions to prepare pupils for the challenges of a digital future.

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Designing individual learning paths through artificial intelligence

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In the context of labor market dynamics and the massive volume of information, the traditional "one-size-fits-all" educational system is increasingly proving its limitations. This article explores how Artificial Intelligence is revolutionizing instructional design by creating individual learning paths. It analyzes the mechanisms through which machine learning algorithms and predictive analytics assess each student's knowledge level, learning style, pace, and specific interests in real time. The paper highlights the major benefits of this approach, such as increasing information retention rates, optimizing study time, and boosting intrinsic motivation. Furthermore, ethical and technical challenges are addressed, including data security and the risk of digital isolation. Finally, the article proposes a conceptual framework for the harmonious integration of AI into the educational ecosystem, emphasizing the redefined role of the teacher—shifting from a mere transmitter of information to a mentor and facilitator of a technology-guided journey.

Keywords *Artificial Intelligence, individual learning path, personalized education, educational technologies (EdTech), adaptive learning.*

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Design and Empirical Validation of a Ninth-Grade Computer Science Textbook Oriented toward the Development of Algorithmic Thinking and the Integration of STEAM Elements

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This paper presents the design, classroom implementation, and empirical validation of a ninth-grade Computer Science textbook developed to support progressive learning, problem solving, and the integration of STEAM elements. The textbook was created in accordance with the 2019 National Curriculum and includes accessible explanations, step-by-step examples, practical tasks, and activities focused on the development of digital products. The study involved 33 ninth-grade students and combined a nine-item Likert-scale questionnaire with open-ended questions. The results indicate a favorable evaluation of the textbook, with an overall mean score of 4.22 out of 5 and very high internal consistency of the instrument (Cronbach's $\alpha = 0.982$). Students particularly appreciated the usefulness of the examples, the practical character of the tasks, and the clear organization of the content. The findings support the use of the textbook as a teaching and learning resource, although they do not allow causal conclusions regarding the development of algorithmic thinking.

Keywords *Algorithmic thinking; Computer Science textbook; STEAM education; empirical validation; digital competences.*

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Leveraging Open Educational Resources in the University Course "Didactics of Mathematics": Challenges and Solutions

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The university course "Didactics of Mathematics," mandatory for the training of both mathematics teachers and primary school teachers, requires modernization at the structural and content levels in the context of recent curricular reforms. A crucial dimension of this modernization is the integration of Open Educational Resources (OER) to foster the professional competencies of future

teachers. Through a documentary and empirical analysis, the article examines the challenges of this integration: the digital native paradox, the epistemological gap (the absence of a historical dimension), the overestimation of interactivity at the expense of conceptual understanding, and the need to adapt OER to the specific didactic context. The proposed solutions are grounded in global best practices but are rigorously adapted to the educational context of the Republic of Moldova, taking into account the specifics of undergraduate training, the requirements of the pre-university curriculum (including the primary cycle), and current educational policy documents.

Keywords*OER in mathematics, didactics of mathematics, training mathematics teachers, training primary school teachers.*

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